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FIVE ALGORITHMS FOR THE EARTH TIDE TERMS IN THE FORCE MODEL OF --ETC(U)
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FIVE ALGORITHMS FOR THE EARTH TIDE TERMS IN THE FORCE MODEL OF NSWC COMPUTER PROGRAMS FOR SATELLITE GEODESY

by

WALTER GROEGER
Strategic Systems Department

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JUL 12 1979

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The most recent versions of five computer algorithms which jointly specify the gravitational action by which the tidal dislocation of the earth's masses perturbs satellite motion are presented.

The first ~~of the five~~ computer routines formulates the perturbing acceleration which the semidiurnal lunar component of the atmospheric tide bulge exerts on the orbit of an artificial earth satellite. The tide bulge is assumed to result from the fact that the earth rotates within the field of lunar mass attraction, the latter being inhomogeneous across the terrestrial globe. (see back)

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The second routine augments the first by spelling out the combined effect of solar heating and solar mass attraction. The semidiurnal term as well as the diurnal term are considered.

The third algorithm accounts for the ocean tide. Only the semidiurnal lunar tide is presently considered, but the algorithm sets the pattern for introducing various other spectral components of the fluid tide, if desired. The algorithm starts from a table for the global tide amplitudes and phase angles. It subsequently approximates the gravitational potential exterior to the earth caused by the tidal bulge by that of a system of point masses located on the ocean surface.

The fourth tide routine is optional; It may be used in place of the third algorithm if one wishes to base the tide potential on a functionalization of the tidal sea surface.

Finally, there is a computer routine for the Newtonian attraction caused by the tide bulge of the solid earth. Both the lunar and solar tide components are included. The tidal properties are assumed to be functions of latitude. Accordingly, Love coefficients are featured that are zonal harmonic expansions of latitude. ~~An~~ allowance has been made for tidal lag. The latter manifests itself as a time delay in the response of the tidal mass redistribution to the tidal stress field. Its origin are the elasticity and plasticity of the earth and the inertia of the masses involved.

FOREWORD

Recent progress in instrumentation for data acquisition and computing methods suggests that satellite geodesy must concern itself with even such minute effects as the gravitational action that the tidal dislocations of the earth's masses exert upon an artificial satellite. A number of computer algorithms suited to this purpose were developed at the Naval Surface Weapons Center (NSWC) during the past few years. The relevant physics background as well as sketches of the computer routines were published in a sequence of four technical reports in 1976 and 1977. Since then, the algorithms were worked out in detail, coded for the computer, and subjected to the customary numerical checkout. Also, an alternate procedure was developed for the ocean tides, which increased the number of tide routines from the original four to five. The necessary derivations for the additional routine were never documented. They are included as Appendix A. The present report is intended to serve as a manual of the five algorithms, each being presented in a form ready for immediate computer implementation, complete with checkout data.

The work documented here was done in the Astronautics and Geodesy Division and was funded as part of the development of the TERRA computer program which is used to determine the earth's gravity field from terrestrial and satellite observations. Henry E. Castro of the Computer Program Division coded those algorithms that involve the atmospheric tides and the two versions of the ocean tide. R. Gordon Barker, also of the Computer Program Division, coded the solid earth tide. Additionally, these two gentlemen invested a great effort while making the required checkout runs on the computer. Raymond B. Manrique of the Astronautics and Geodesy Division and the author jointly outlined the computer algorithms, with the author bearing the overall responsibility for the finished formulations and the sole responsibility for the numerical checkout calculations.

This report was reviewed by Richard J. Anderle, Head, Astronautics and Geodesy Division.

Released by:

Ralph A. Niemann

R. A. NIEMANN, Head
Strategic Systems Department

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INTRODUCTION

Several years ago, while compiling a force model on which to base the trajectory integration in the TERRA[1] and CELEST[2] computer programs for satellite geodesy and satellite ephemeris computation, the author faced the nearly complete lack of published algorithms that would in a suitable form express the gravitational action by which the tidal dislocation of the earth's masses perturbs satellite motion. At the same time, it was realized that the earth tides would have to be taken into account if TERRA and CELEST were to substantially exceed the accuracy requirements of their forerunners, GEO[3] and ASTRO.[4] An effort was consequently made to derive the necessary disturbing potentials and perturbing accelerations. The result consisted of a number of mathematical models, each designed to be realistic and describing one type of perturbation corresponding to one of the four earth tides. The details of these models were documented in four technical papers.[5,6,7,8] Simultaneously, the necessary computer routines were developed and submitted for coding.

The first tide routine formulated the perturbing acceleration which the semidiurnal lunar component of the atmospheric tide bulge exerts on the orbit of an artifical satellite. The tide bulge was assumed to result from the fact that the earth rotates within the field of lunar mass attraction which is inhomogeneous across the terrestrial globe. The routine was restricted to the main term of the semidiurnal tide.

The second tide routine was designed to augment the first by spelling out the combined effect on the atmosphere of solar heating and solar mass attraction, plus the resulting modification of the earth's gravitation. The semidiurnal term as well as the diurnal term were included.

The third algorithm was to account for the ocean tide. It started from an existing table for the global tide amplitudes and phase angles, and postulated that each tidal height value resembles a gravitating point mass that by its presence perturbs the satellite motion. Only the semidiurnal lunar tide was considered; however, this algorithm may be regarded as setting the pattern for introducing various other spectral components of the fluid tide, if desired.

Further, there was a computer routine for the Newtonian attraction caused by the tide bulge of the solid earth. Both the lunar and the solar tide components were included. The tidal properties were assumed to be functions of latitude. Accordingly, Love coefficients were featured that are zonal harmonic expansions of latitude. An allowance was made for tidal lag manifesting itself as a time delay in the response of the tidal mass redistribution to the tidal stress field.

Several months after completing the coding requests for these four algorithms, an opportunity presented itself to devise an interesting alternative to the just mentioned ocean tide routine. Namely, a functionalization became available for the tidal ocean surface. This consisted of a representation of the tidal ocean surface by an expansion, in surface harmonics, that had been least squares fit adjusted to the table of discrete tide amplitudes and phases that had been the basis of the ocean tide algorithm. Upon receiving a card deck containing the coefficients for this expansion [made available to us through the kindness of Dr. C. C. Goad of the National Oceanic and Atmospheric Administration (NOAA)], an optional routine was written, which interprets the functionalized ocean surface as a continuous, gravitating, tidal mass layer. From it, the perturbing acceleration acting on the satellite, due to the presence of the oceanic tide bulge, is then derived via the Poisson integral and subsequent gradient formation. While the physics background and the mathematical derivations for the original four tide routines are available in the shape of formal reports,[5,6,7,8] the background for this latter optional version of the ocean tide algorithm had never before been documented. Thus, to preserve it, it is included in this report as Appendix A.

1 – COMPUTER ROUTINE FOR THE PERTURBING ACCELERATION CAUSED BY THE LUNAR AIR TIDE

INPUT DATA

R = any reasonable value for the mean radius of the earth, in meters

G = gravitational constant, in meters, kilograms, and seconds
suggested value: $G = 6.6732E-11 \ m^3 \ kg^{-1} \ sec^{-2}$

A_2 = a constant associated with the physics of the lunar air tide
suggested value: $A_2 = 0.564 \ kg \ m^{-2}$

The parameters J, n, and t* specify the time instant for which the perturbing acceleration due to the tide is to be found. Normally, this is the time instant associated with the current ***time line*** of the orbit integration.

J = number of the calendar year

n = number of the day within the year

t^* = mean solar time at Greenwich (GMT, UTC), in seconds

$\left.\begin{matrix} x_1 \\ x_2 \\ x_3 \end{matrix}\right\}=$ inertial, Cartesian components of satellite position vector, in meters, associated with the just specified time instant (for a precise definition of the reference frame, see Appendices B and C of Reference 2)

ALGORITHM

$$\delta(i,j) = \begin{cases} 1 & \text{if} \quad i = j \\ 0 & \phantom{\text{if}} \quad i \neq j \end{cases} \qquad (1\text{-}1)$$

$$N = n\,\delta(J,\ 1975) + (365 + n)\,\delta(J,\ 1976) + (731 + n)\,\delta(J,\ 1977) + (1096 + n)\,\delta(J,\ 1978) + (1461 + n)\,\delta(J,\ 1979) + (1826 + n)\,\delta(J,\ 1980) \qquad (1\text{-}2)$$

$$\Delta T = 5.28E - 04 + (3.56E - 08)N \qquad (1\text{-}3)$$

$$d = 27392.5 + N + \Delta T + t^* \qquad (1\text{-}4)$$

$$T = d/36525 \qquad (1\text{-}5)$$

$$s = 270.434358 + 481267.883141T - 0.001133T^2 + 0.000002T^3 \qquad (1\text{-}6)$$

$$h = 279.69668 + 36000.768930T + 0.000303T^2 \qquad (1\text{-}7)$$

s and h will result in decimal fractions of degrees.

Calculate now the earth-fixed, Cartesian satellite coordinates (y_1, y_2, and y_3) from the corresponding inertial coordinates (x_1, x_2, and x_3).

$$\begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = (ABCD)_{J,n,t^*} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \qquad (1\text{-}8)$$

where $(ABCD)_{J,n,t^*}$ is the transformation matrix that manages the transition from the inertial frame of the satellite equations of motion (***Basic Inertial System***, which is associated with either 1950.0 or with the beginning of the day UT of trajectory epoch) to the earth-fixed reference frame corresponding to the time instant, J, n, t*. This transformation is a computer routine that is already part of TERRA/CELEST. It is external to the present algorithm. For details, see Appendices B and C of Reference 2.

In the earth-fixed frame, the components of the perturbing acceleration are

$$T_{y_1} = \frac{\partial U}{\partial r}\frac{\partial r}{\partial y_1} + \frac{\partial U}{\partial \lambda}\frac{\partial \lambda}{\partial y_1} + \frac{\partial U}{\partial \theta}\frac{\partial \theta}{\partial y_1} \tag{1-9}$$

$$T_{y_2} = \frac{\partial U}{\partial r}\frac{\partial r}{\partial y_2} + \frac{\partial U}{\partial \lambda}\frac{\partial \lambda}{\partial y_2} + \frac{\partial U}{\partial \theta}\frac{\partial \theta}{\partial y_2} \tag{1-10}$$

$$T_{y_3} = \frac{\partial U}{\partial r}\frac{\partial r}{\partial y_3} + \frac{\partial U}{\partial \lambda}\frac{\partial \lambda}{\partial y_3} + \frac{\partial U}{\partial \theta}\frac{\partial \theta}{\partial y_3} \tag{1-11}$$

where

$$\frac{\partial r}{\partial y_i} = \frac{y_i}{r} \quad , \quad i = 1, 2, 3 \tag{1-12}$$

$$\frac{\partial \lambda}{\partial y_1} = - \frac{y_2}{y_1^2 + y_2^2} \tag{1-13}$$

$$\frac{\partial \lambda}{\partial y_2} = \frac{y_1}{y_1^2 + y_2^2} \tag{1-14}$$

$$\frac{\partial \lambda}{\partial y_3} = 0 \tag{1-15}$$

$$\frac{\partial \theta}{\partial y_1} = \frac{-y_1 \, y_3}{r^2 \sqrt{y_1^2 + y_2^2}} \tag{1-16}$$

$$\frac{\partial \theta}{\partial y_2} = \frac{- y_2 \, y_3}{r^2 \sqrt{y_1^2 + y_2^2}} \tag{1-17}$$

$$\frac{\partial \theta}{\partial y_3} = \frac{\sqrt{y_1^2 + y_2^2}}{r^2} \tag{1-18}$$

$$\frac{\partial U}{\partial r} = -a\frac{3R^3}{r^4}P_2^2(\sin\theta)\ \cos 2\alpha + b\frac{5R^5}{r^6}P_4^2(\sin\theta)\ \cos 2\alpha \tag{1-19}$$

$$\frac{\partial U}{\partial \lambda} = -a\frac{2R^3}{r^3}P_2^2(\sin\theta)\ \sin 2\alpha + b\frac{2R^5}{r^5}P_4^2(\sin\theta)\ \sin 2\alpha \tag{1-20}$$

$$\frac{\partial U}{\partial \theta} = -a\frac{3R^3}{r^3}\sin 2\theta\ \cos 2\alpha + b\frac{15R^5}{r^5}(7\sin^2\theta - 4)\ \sin 2\theta\ \cos 2\alpha \tag{1-21}$$

$$r = +\sqrt{y_1^2 + y_2^2 + y_3^2} \tag{1-22}$$

$$a = A_2 GR\frac{5\pi^2}{64} \tag{1-23}$$

$$b = A_2 GR\frac{5\pi^2}{3072} = \frac{a}{48} \tag{1-24}$$

$$P_2^2(x) = 3(1 - x^2) \tag{1-25}$$

$$P_4^2(x) = \frac{15}{2}(1 - x^2)(7x^2 - 1) \tag{1-26}$$

$$x = \sin\theta = \frac{y_3}{r} \tag{1-27}$$

$$\sin 2\theta = \frac{2y_3\sqrt{y_1^2 + y_2^2}}{r^2} \tag{1-28}$$

$$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha \tag{1-29}$$

$$\sin 2\alpha = 2\sin\alpha\ \cos\alpha \tag{1-30}$$

$$\cos\alpha = \cos\alpha^*\ \cos\lambda - \sin\alpha^*\ \sin\lambda \tag{1-31}$$

$$\sin\alpha = \sin\alpha^*\ \cos\lambda + \cos\alpha^*\ \sin\lambda \tag{1-32}$$

$$\alpha^* = t^{**} - \nu - 7.^\circ 5 \tag{1-33}$$

$$t^{**} = \frac{360}{86400}t^* \tag{1-34}$$

$$\nu = s - h \tag{1-35}$$

$$\cos\lambda = \frac{y_2}{\sqrt{y_1^2 + y_2^2}} \tag{1-36}$$

$$\sin \lambda = \frac{y_2}{\sqrt{y_1^2 + y_2^2}} \tag{1-37}$$

Finally, transform the perturbing acceleration back into the inertial frame

$$\begin{pmatrix} T_{x_1} \\ T_{x_2} \\ T_{x_3} \end{pmatrix} = (ABCD)^T_{J,n,t*} \begin{pmatrix} T_{y_1} \\ T_{y_2} \\ T_{y_3} \end{pmatrix} \tag{1-38}$$

where $(ABCD)^T$ is the transpose of (ABCD).

COMPUTER PROGRAM OUTPUT

The inertial components (T_{x_1}, T_{x_2}, and T_{x_3}) of the perturbing acceleration due to the presence of the lunar air tide are output in terms of m sec^{-2}. If required, convert to km sec^{-2}.

TRIAL DATA FOR PROGRAM CHECKOUT

$R = 6378145$	m
$G = 6.6732E - 11$	$m^3\ kg^{-1}\ sec^{-2}$
$A_2 = 0.564$	$kg\ m^{-2}$
$J = 77$	
$n = 202$	
$t^* = 50000$	sec
$x_1 = 3151529.23$	m
$x_2 = 5458608.75$	m
$x_3 = 3639072.50$	m

From external source (CELEST subroutine PRENUT)

$$(ABCD) = \begin{pmatrix} -0.8405285753 & 0.5417623775 & 0.2289080162E-02 \\ -0.5417605355 & -0.8405316908 & 0.1413662999E-02 \\ 0.2689913850E-02 & -0.5190827376E-04 & 0.9999963803 \end{pmatrix}$$

$y_1 = +316648.61$ m

$y_2 = -6290363.38$ m

$y_3 = +3647253.32$ m

$t^{**} = 208\overset{\circ}{.}3333333$

$N = (731 + 202) = 933$

$\Delta T = 5.612148000E-04$

$t^{*} = 0.5787037037$ deg

$d = 2.832607926E+04$

$T = 0.7755257840$

$h = 2\overset{\circ}{.}819922141E+04$

$s = 3\overset{\circ}{.}735060861E+05$

$\lambda = 272\overset{\circ}{.}8817616$

$h - s = -3\overset{\circ}{.}4530686469E+05$

$\alpha = -3\overset{\circ}{.}448331496E+05$

$2\alpha = -6\overset{\circ}{.}896662992E+05$

$x = 0.5011240256$

$P_2^2(x) = 2.246624133$

$P_4^2(x) = 4.256662025$

$$\frac{\partial U}{\partial r} = 7.070049216E-12 \qquad \text{m sec}^{-2}$$

$$\frac{\partial U}{\partial \lambda} = -5.416307419E-04 \qquad \text{m}^2 \text{ sec}^{-2}$$

$$\frac{\partial U}{\partial \theta} = 2.467494681E-05 \qquad \text{m}^2 \text{ sec}^{-2}$$

$$\frac{\partial r}{\partial y_1} = 4.350677408E-02$$

$$\frac{\partial \lambda}{\partial y_1} = 1.585715104E-07 \qquad \text{m}^{-1}$$

$$\frac{\partial \theta}{\partial y_1} = -3.461599518E-09 \qquad \text{m}^{-1}$$

$$\frac{\partial r}{\partial y_2} = -8.642811299E-01$$

$$\frac{\partial \lambda}{\partial y_2} = 7.982281040E-09 \qquad \text{m}^{-1}$$

$$\frac{\partial \theta}{\partial y_2} = 6.876619114E-08 \qquad \text{m}^{-1}$$

$$\frac{\partial r}{\partial y_3} = 5.011240258E-01$$

$$\frac{\partial \lambda}{\partial y_3} = 0$$

$$\frac{\partial \theta}{\partial y_3} = 1.189005543E-07 \qquad \text{m}^{-1}$$

$$T_{y_1} = -8.566502457E-11 \qquad \text{m sec}^{-2}$$

$$T_{y_2} = -8.737156821E-12 \qquad \text{m sec}^{-2}$$

$T_{y_3} = 6.476836379E-12 \quad m\ sec^{-2}$

$T_{x_1} = 7.675476994E-11 \quad m\ sec^{-2}$
$\phantom{T_{x_1}} = 7.675476994E-14 \quad km\ sec^{-2}$

$T_{x_2} = -3.906656638E-11 \quad m\ sec^{-2}$
$\phantom{T_{x_2}} = -3.906656638E-14 \quad km\ sec^{-2}$

$T_{x_3} = 6.268367431E-12 \quad m\ sec^{-2}$
$\phantom{T_{x_3}} = 6.268367431E-15 \quad km\ sec^{-2}$

$T = |\overline{T}| = 8.635267078E-11 \quad m\ sec^{-2}$
$\phantom{T = |\overline{T}|} = 8.635267078E-14 \quad km\ sec^{-2}$

2 – COMPUTER ROUTINE FOR THE PERTURBING ACCELERATION CAUSED BY THE SOLAR AIR TIDE

INPUT DATA

R = any reasonable value for the mean radius of the earth, in meters
G = gravitational constant, in meters, kilograms, and seconds
suggested value: $G = 6.6732E-11\ m^3\ kg^{-1}\ sec^{-2}$

$\left.\begin{matrix} A_1 \\ \\ A_2 \end{matrix}\right\}$ = constants associated with the physics of the solar air tide
suggested values: $A_1 = 6\ kg\ m^{-2}$
$A_2 = 11.9\ kg\ m^{-2}$

The parameters J, n, and t* specify the time instant for which the perturbing acceleration due to the tide is to be found. Normally, this is the time instant associated with the current ***time line*** of the orbit integration.

J = number of the calendar year
n = number of the day within the year
t* = mean solar time at Greenwich (GMT, UTC), in seconds

$\left.\begin{matrix} x_1 \\ x_2 \\ x_3 \end{matrix}\right\}$ = inertial, Cartesian components of the satellite position vector, in meters, associated with the just specified time instant (for a precise definition of the reference frame, see Appendices B and C of Reference 2)

ALGORITHM

First, calculate the earth-fixed, Cartesian satellite coordinates (y_1, y_2, and y_3) from the corresponding inertial coordinates (x_1, x_2, and x_3).

$$\begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = (ABCD)_{J,n,t^*} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \tag{2-1}$$

where $(ABCD)_{J,n,t^*}$ is the transformation matrix that manages the transition from the inertial frame of the satellite equations of motion (***Basic Inertial System***, which is associated with either 1950.0 or with the beginning of the day UT of trajectory epoch) to the earth-fixed reference frame corresponding to the time instant, J, n, t*. This transformation is a computer routine that is already part of TERRA/CELEST. It is external to the present algorithm. For details, see Appendices B anc C of Reference 2.

In the earth-fixed frame, the components of the perturbing acceleration are

$$T_{y_1} = \frac{\partial U}{\partial r}\frac{\partial r}{\partial y_1} + \frac{\partial U}{\partial \lambda}\frac{\partial \lambda}{\partial y_1} + \frac{\partial U}{\partial \theta}\frac{\partial \theta}{\partial y_1} \tag{2-2}$$

$$T_{y_2} = \frac{\partial U}{\partial r}\frac{\partial r}{\partial y_2} + \frac{\partial U}{\partial \lambda}\frac{\partial \lambda}{\partial y_2} + \frac{\partial U}{\partial \theta}\frac{\partial \theta}{\partial y_2} \tag{2-3}$$

$$T_{y_3} = \frac{\partial U}{\partial r}\frac{\partial r}{\partial y_3} + \frac{\partial U}{\partial \lambda}\frac{\partial \lambda}{\partial y_3} + \frac{\partial U}{\partial \theta}\frac{\partial \theta}{\partial y_3} \tag{2-4}$$

where

$$\frac{\partial r}{\partial y_i} = \frac{y_i}{r} \quad , \quad i = 1,2,3 \tag{2-5}$$

$$\frac{\partial \lambda}{\partial y_1} = - \frac{y_2}{y_1^2 + y_2^2} \tag{2-6}$$

$$\frac{\partial \lambda}{\partial y_2} = \frac{y_1}{y_1^2 + y_2^2} \tag{2-7}$$

$$\frac{\partial \lambda}{\partial y_3} = 0 \tag{2-8}$$

$$\frac{\partial \theta}{\partial y_1} = \frac{-y_1 y_3}{r^2\sqrt{y_1^2 + y_2^2}} \tag{2-9}$$

$$\frac{\partial \theta}{\partial y_2} = \frac{-y_2 y_3}{r^2\sqrt{y_1^2 + y_2^2}} \tag{2-10}$$

$$\frac{\partial \theta}{\partial y_3} = \frac{\sqrt{y_1^2 + y_2^2}}{r^2} \tag{2-11}$$

$$\begin{aligned}\frac{\partial U}{\partial r} &= a_1 \frac{4R^4}{r^5} P_3^1(\sin\theta)\ \cos\alpha - a_2 \frac{3R^3}{r^4} P_2^2(\sin\theta)\ \cos 2\beta \\ &\quad + a_3 \frac{5R^5}{r^6} P_4^2(\sin\theta)\ \cos 2\beta\end{aligned} \tag{2-12}$$

$$\begin{aligned}\frac{\partial U}{\partial \lambda} &= a_1 \frac{R^4}{r^4} P_3^1(\sin\theta)\ \sin\alpha - a_2 \frac{2R^3}{r^3} P_2^2(\sin\theta)\ \sin 2\beta \\ &\quad + a_3 \frac{2R^5}{r^5} P_4^2(\sin\theta)\ \sin 2\beta\end{aligned} \tag{2-13}$$

$$\begin{aligned}\frac{\partial U}{\partial \theta} &= a_1 \frac{3R^4}{2r^4} (15 \sin^2\theta - 11)\ \sin\theta\ \cos\alpha - a_2 \frac{3R^3}{r^3} \sin 2\theta\ \cos 2\beta \\ &\quad + a_3 \frac{15R^5}{r^5} (7\sin^2\theta - 4)\ \sin 2\theta\ \cos 2\beta\end{aligned} \tag{2-14}$$

$$r = +\sqrt{y_1^2 + y_2^2 + y_3^2} \tag{2-15}$$

$$a_1 = A_1 \frac{8\pi GR}{105} \tag{2-16}$$

$$a_2 = A_2 \frac{5\pi^2 GR}{64} \tag{2-17}$$

$$a_3 = A_2 \frac{5\pi^2 GR}{3072} = \frac{a_2}{48} \tag{2-18}$$

$$P_3^1(x) = \frac{3}{2}\sqrt{1 - x^2}(5x^2 - 1) \tag{2-19}$$

$$P_2^2(x) = 3(1 - x^2) \tag{2-20}$$

$$P_4^2(x) = \frac{15}{2}(1 - x^2)(7x^2 - 1) \tag{2-21}$$

$$x = \sin\theta = \frac{y_3}{r} \tag{2-22}$$

$$\sin 2\theta = \frac{2y_3\sqrt{y_1^2 + y_2^2}}{r^2} \tag{2-23}$$

$$\cos\alpha = \cos\alpha^* \cos\lambda - \sin\alpha^* \sin\lambda \tag{2-24}$$

$$\sin\alpha = \cos\alpha^* \sin\lambda + \sin\alpha^* \cos\lambda \tag{2-25}$$

$$\cos 2\beta = \cos^2\beta - \sin^2\beta \tag{2-26}$$

$$\sin 2\beta = 2\sin\beta\cos\beta \tag{2-27}$$

$$\cos\beta = \cos\beta^* \cos\lambda - \sin\beta^* \sin\lambda \tag{2-28}$$

$$\sin\beta = \sin\beta^* \cos\lambda + \cos\beta^* \sin\lambda \tag{2-29}$$

$$\cos\lambda = \frac{y_1}{\sqrt{y_1^2 + y_2^2}} \tag{2-30}$$

$$\sin\lambda = \frac{y_2}{\sqrt{y_1^2 + y_2^2}} \tag{2-31}$$

$$\alpha^* = t^{**} - 78^\circ \tag{2-32}$$

$$\beta^* = t^{**} - 146^\circ \tag{2-33}$$

$$t^{**} = \frac{360}{86400} t^* \tag{2-34}$$

Finally, transform the perturbing acceleration back into the inertial frame

$$\begin{pmatrix} T_{x_1} \\ T_{x_2} \\ T_{x_3} \end{pmatrix} = (ABCD)^T_{J,n,t^*} \begin{pmatrix} T_{y_1} \\ T_{y_2} \\ T_{y_3} \end{pmatrix} \tag{2-35}$$

where $(ABCD)^T$ is the transpose of (ABCD).

While coding the above algorithm for use with TERRA and CELEST, it was found helpful to keep in mind the following. A time line is characterized by the time elapsed since the epoch (starting instant) of the particular orbital arc. The epoch is specified in terms of year, number of the day UT in the year, and time (t_{EP}), in seconds, elapsed from the start of the day UT until epoch time. Usually, $t_{EP} = 0$. The present algorithm is periodic in t^*, the latter being the Universal time associated with the individual time line. t^* starts from zero at the beginning of the day UT during which the time line occurs. It is thus unimportant how many integer days have elapsed since epoch. For an illustration, see Figure 1.

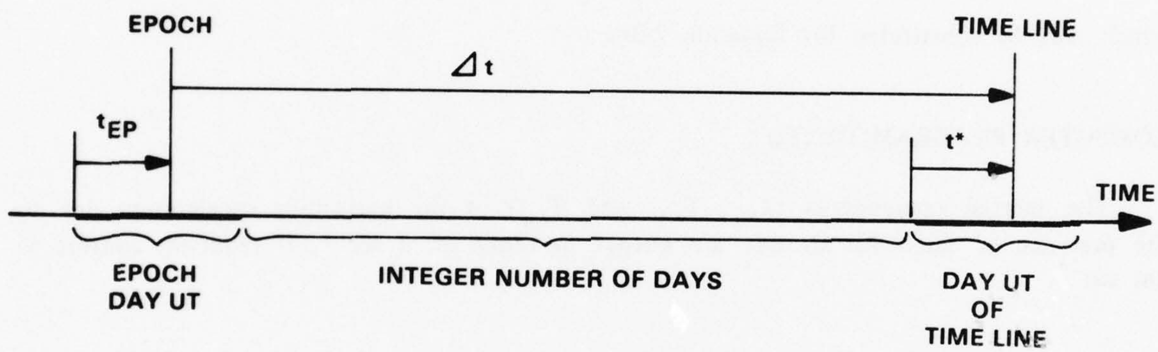


Figure 1. Universal Time in the Algorithm for the Perturbing Acceleration Due to the Solar Air Tide

To actually find the numerical value of t^* corresponding to the time line, consider that

t_{EP} = Epoch time (Universal time at epoch)
Δt = time difference between time line and epoch
t^* = Universal time of time line

Now, express t_{EP} (if non-zero) and Δt and t^* in terms of a common time unit (usually mean solar seconds). Find

$$t' = t_{EP} + \Delta t \qquad (2\text{-}36)$$

Express t' in terms of days and decimal fraction of days (1 day = 86400 sec). Now,

$$t^* = \mathrm{FRACT}(t') \qquad (2\text{-}37)$$

Finally, convert t^* to angular time

$$t^{**} = 360° \ \mathrm{FRACT}(t') \qquad (2\text{-}38)$$

which may be substituted for Equation 2-34.

COMPUTER PROGRAM OUTPUT

The inertial components (T_{x_1}, T_{x_2}, and T_{x_3}) of the perturbing acceleration due to the presence of the solar air tide are output in terms of m sec^{-2}. If required, convert to km sec^{-2}.

TRIAL DATA FOR PROGRAM CHECKOUT

$R = 6378145$	m
$G = 6.6732E-11$	$m^3\ kg^{-1}\ sec^{-2}$
$A_1 = 6$	$kg\ m^{-2}$
$A_2 = 11.9$	$kg\ m^{-2}$
$J = 1977$	
$n = 202$	
$t^* = 50000$	sec

x_1 = 3151529.23 m

x_2 = 5458608.75 m

x_3 = 3639072.50 m

From external source (CELEST subroutine PRENUT)

$$(ABCD) = \begin{pmatrix} -0.8405285753 & 0.5417623775 & 0.2289080162E-02 \\ -0.5417605355 & -0.8405316908 & 0.1413662999E-02 \\ 0.2689913850E-02 & -0.5190827376E-04 & 0.9999963803 \end{pmatrix}$$

y_1 = 316648.61 m

y_2 = - 6290363.38 m

y_3 = 3647253.32 m

r = 7278145.00 m

λ = $272^\circ.8817616$

θ = $30^\circ.07439285$

t^{**} = $208^\circ.3333333$

α = $403^\circ.215095$

β = $335^\circ.215095$

$P_3^1(x)$ = 0.3318192840

$P_2^2(x)$ = 2.246624133

$P_4^2(x)$ = 4.256662025

a_1 = 6.112661413E - 04 $m^2\ sec^{-2}$

a_2 = 3.905397704E - 03 $m^2\ sec^{-2}$

a_3 = 8.136245217E - 05 $m^2\ sec^{-2}$

$$\frac{\partial U}{\partial r} = -1.450823033E-09 \qquad m\ sec^{-2}$$

$$\frac{\partial U}{\partial \lambda} = 8.799066591E-03 \qquad m^2\ sec^{-2}$$

$$\frac{\partial U}{\partial \theta} = -6.659222442E-03 \qquad m^2\ sec^{-2}$$

$$= 4.350677408E-02$$

$$_{1} \quad = 1.585715104E-07 \qquad m^{-1}$$

$$\frac{\partial \theta}{\partial y_1} = -3.461599518E-09 \qquad m^{-1}$$

$$\frac{\partial r}{\partial y_2} = -8.642811299E-01$$

$$\frac{\partial \lambda}{\partial y_2} = 7.982281040E-09 \qquad m^{-1}$$

$$\frac{\partial \theta}{\partial y_2} = 6.876619116E-08 \qquad m^{-1}$$

$$\frac{\partial r}{\partial y_3} = 5.011240258E-01$$

$$\frac{\partial \lambda}{\partial y_3} = 0$$

$$\frac{\partial \theta}{\partial y_3} = 1.189005543E-07 \qquad m^{-1}$$

$$T_{y_1} = 1.355212210E-09 \qquad m\ sec^{-2}$$

$$T_{y_2} = 8.662262286E-10 \quad m\ sec^{-2}$$

$$T_{y_3} = -1.518827519E-09 \quad m\ sec^{-2}$$

$$T_{x_1} = -1.612467289E-09 \quad m\ sec^{-2}$$

$$= -1.612467289E-12 \quad km\ sec^{-2}$$

$$T_{x_2} = 6.191232115E-12 \quad m\ sec^{-2}$$

$$= 6.191232115E-15 \quad km\ sec^{-2}$$

$$T_{x_3} = -1.514495280E-09 \quad m\ sec^{-2}$$

$$= -1.514495280E-12 \quad km\ sec^{-2}$$

$$T = |\bar{T}| = 2.212190101E-09 \quad m\ sec^{-1}$$

$$= 2.212190101E-12 \quad km\ sec^{-2}$$

3 – FIRST COMPUTER ROUTINE FOR THE PERTURBING ACCELERATION CAUSED BY THE M_2 COMPONENT OF THE OCEAN TIDE (BASED ON A DISCRETE REPRESENTATION OF THE SEA SURFACE)

INPUT DATA

ζ_{ij} = tidal amplitude on area element ij (these are output data from the NSWC/Schwiderski Ocean Tide Program); if available in meters, use directly; otherwise, convert to meters

δ_{ij} = tidal phase angle on area element ij (these are output data from the NSWC/Schwiderski Ocean Tide Program), available in terms of degrees

NP = number of grid points in the NSWC/Schwiderski Ocean Tide Program; this may be expected to be a five-digit integer

R = ***radius of the earth*** (semimajor axis of a suitable reference ellipsoid), in kilometers
suggested value: R = 6378.145 km

ϵ^2 = square of eccentricity of reference ellipsoid
suggested value: $\epsilon^2 = 0.00669342$
in case it is desired to start from the flattening, f, find ϵ^2 from $\epsilon^2 = (2 - f)f$

μ_E = gravitational constant of the earth, in kilometers and seconds
suggested value: $\mu_E = 398601 \text{ km}^3 \text{ sec}^{-2}$

G = Newton's constant, in kilometers, kilograms, and seconds
suggested value: $G = 6.6732E - 20 \text{ km}^3 \text{ kg}^{-1} \text{ sec}^{-2}$

ρ = density of water, in kilograms and kilometers
suggested value: $\rho = 1.E + 12 \text{ kg km}^{-3}$

σ = rate of mean mean longitude of moon
suggested value: $\sigma = \frac{180}{\pi} 1.40519E - 04 \text{ deg sec}^{-1}$

The parameters J, n, and t* specify the time instant for which the perturbing acceleration due to the tide is to be found. Normally, this is the time instant associated with the current ***time line*** of the orbit integration

J = number of the calendar year
n = number of the day within the year
t* = mean solar time at Greenwich (GMT, UTC), in seconds

$\left.\begin{matrix} x_1 \\ x_2 \\ x_3 \end{matrix}\right\}$ = inertial, Cartesian components of satellite position vector, in kilometers, associated with the just specified time instant (for a precise definition of the reference frame, see Appendices B and C of Reference 2)

NMAX = limit on the number of terms in the expansion of the tide potential

SUBROUTINE FOR THE TIDE POTENTIAL COEFFICIENTS

This is the first part of the algorithm for the first ocean tide computer routine. The time-independent constituents of the expansion coefficients of the ocean tide potential (${}^{\alpha}F_{nm}$, ${}^{\beta}F_{nm}$, ${}^{\alpha}H_{nm}$, and ${}^{\beta}H_{nm}$) are calculated from the amplitude and phase tables produced by the NSWC/Schwiderski Ocean Tide Computer Program. Once established, the latter remain unchanged as long as the current versions of the amplitude and phase tables remain valid. Thus, the subroutine under discussion will be exercised quite infrequently, namely, just once each time a new edition of the just mentioned amplitude and phase

tables becomes available. That is expected to occur at intervals of several years, during which the present subroutine will remain dormant.

For the geometry associated with the index ij, see Figures 2, 3, and 4.

$$\alpha_{ij} = 10^{-3} \rho\, G\, \Delta S_{ij}\, \zeta_{ij} \cos \delta_{ij} \tag{3-1}$$

$$\beta_{ij} = 10^{-3} \rho\, G\, \Delta S_{ij}\, \zeta_{ij} \sin \delta_{ij} \tag{3-2}$$

$$\Delta S_{ij} = \left(\frac{\pi}{180}\right)^2 R^2 \sin\left(\frac{\pi}{180} j\right) \tag{3-3}$$

= area of the nonpolar surface area element for $j = 2,3,4 \cdots, j_{MAX} < 180$

$$(\Delta S_{ij})_{POLAR} = \frac{1}{2}\left(\frac{\pi}{180}\right)^3 R^2 \tag{3-4}$$

= area of the polar surface area element (neighboring the North Pole)

$$\theta_{ij} = \frac{\pi}{2} - \frac{\pi}{180}\left(j - \frac{1}{2}\right) \quad \text{in radians} \tag{3-5}$$

$$\lambda_{ij} = \frac{\pi}{180}\left(i - \frac{1}{2}\right) \quad \text{in radians} \tag{3-6}$$

$$\rho_{ij} = \left(1 - \frac{\epsilon^2}{2} \sin^2 \theta_{ij}\right) \tag{3-7}$$

To each index ij assign now a counting number ν, $1 \leqslant \nu \leqslant NP$.

$$^{\alpha}F_{nm} = (2 - \delta^0_m)\frac{(n-m)!}{(n+m)!}\frac{1}{\mu_E R^{2n}} \sum_{\nu=1}^{NP} \rho_\nu^{2n+1} \alpha_\nu f^\nu_{nm} \tag{3-8}$$

$$^{\beta}F_{nm} = (2 - \delta^0_m)\frac{(n-m)!}{(n+m)!}\frac{1}{\mu_E R^{2n}} \sum_{\nu=1}^{NP} \rho_\nu^{2n+1} \beta_\nu f^\nu_{nm} \tag{3-9}$$

$$^{\alpha}H_{nm} = \frac{(n-m)!}{(n+m)!}\frac{1}{\mu_E R^{2n}} \sum_{\nu=1}^{NP} \rho_\nu^{2n+1} \alpha_\nu h^\nu_{nm} \tag{3-10}$$

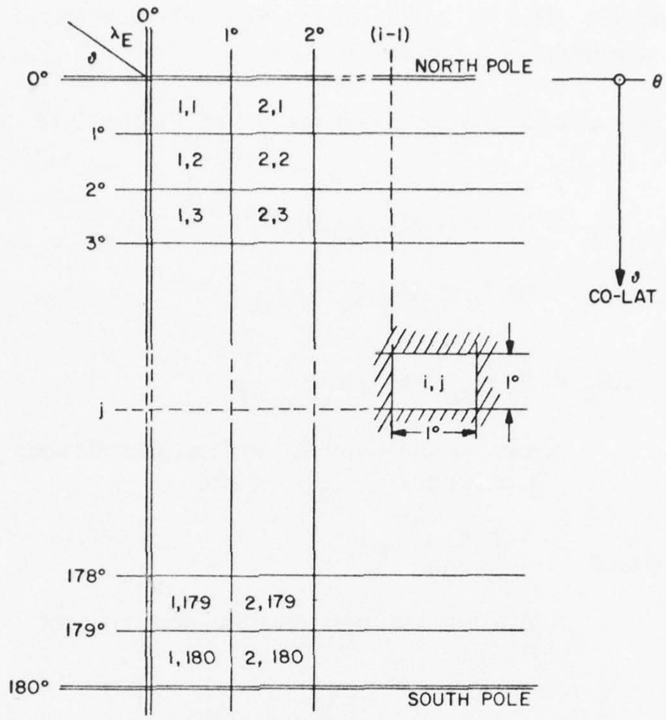


Figure 2. Division of the Earth's Surface Into Area Elements According to the Schwiderski Ocean Tide Model

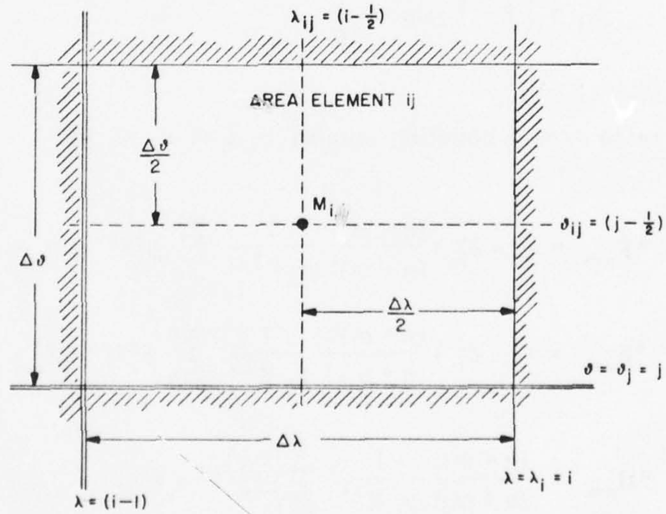
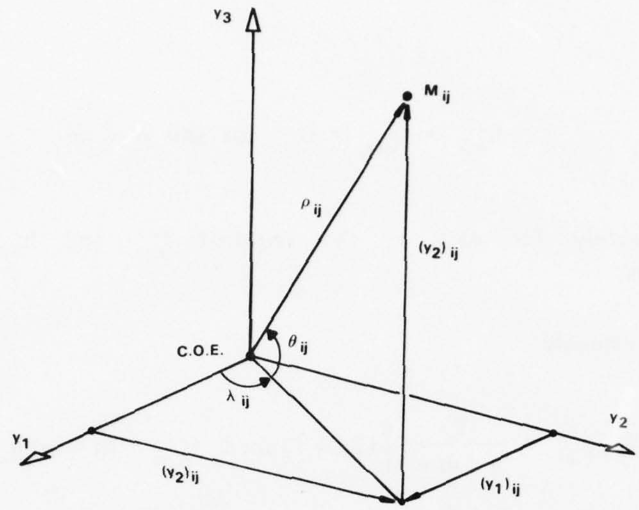


Figure 3. Position of Point Mass at the Geometrical Center of the Surface Area Element Associated With ij



y_1 = INDICATES THE POINT WHERE GREENWICH MERIDIAN AND EQUATOR INTERSECT

y_2 = LOCATED IN THE EQUATORIAL PLANE

Figure 4. Position of the Point Mass M_{ij} in the Earth-Fixed Cartesian Coordinate Frame

$$ {}^{\beta}H_{nm} = \frac{(n-m)!}{(n+m)!} \frac{1}{\mu_E R^{2n}} \sum_{\nu=1}^{NP} \rho_\nu^{2n+1} \beta_\nu h_{nm}^\nu \tag{3-11} $$

$$ \delta_k^\ell = \begin{cases} 1 & \text{if} \quad \ell = k \\ 0 & \phantom{\text{if}} \quad \ell \neq k \end{cases} \tag{3-12} $$

Calculate the f_{nm}^ν and h_{nm}^ν as follows, noting that

$$ \rho_\nu = \frac{R}{\rho_\nu} \tag{3-13} $$

and

$$ h_{no}^\nu = 0 \quad \text{for all values of n} \tag{3-14} $$

and that

$$h^{\nu}_{nm} = f^{\nu}_{nm} = 0 \quad \text{for any } n < m \tag{3-15}$$

Obtain, separately for each ν, the required f^{ν}_{nm} and h^{ν}_{nm} from the following recurrence relations:

To advance in n, evaluate

$$f^{\nu}_{n+1,m} = \frac{p_\nu}{n - m + 1}\left[(2n + 1) \sin \theta_\nu f^{\nu}_{n,m} - (n + m) p_\nu f^{\nu}_{n-1,m}\right] \tag{3-16}$$

$$h^{\nu}_{n+1,m} = \frac{p_\nu}{n - m + 1}\left[(2n + 1) \sin \theta_\nu h^{\nu}_{n,m} - (n + m) p_\nu h^{\nu}_{n-1,m}\right] \tag{3-17}$$

To advance in m, use

$$f^{\nu}_{n+1,n+1} = (2n + 1)p_\nu\left[\cos \theta_\nu \cos \lambda_\nu f^{\nu}_{n,n} - \cos \theta_\nu \sin \lambda_\nu h^{\nu}_{n,n}\right] \tag{3-18}$$

$$h^{\nu}_{n+1,n+1} = (2n + 1)p_\nu\left[\cos \theta_\nu \cos \lambda_\nu h^{\nu}_{n,n} + \cos \theta_\nu \sin \lambda_\nu f^{\nu}_{n,n}\right] \tag{3-19}$$

Start these recurrences from

$$f^{\nu}_{0,0} = \frac{1}{\rho_\nu} \tag{3-20}$$

$$f^{\nu}_{1,0} = R\,\frac{\sin \theta_\nu}{\rho_\nu^2} \tag{3-21}$$

$$h^{\nu}_{0,0} = h^{\nu}_{1,0} = 0 \tag{3-22}$$

and terminate the procedure at n = NMAX.

Store the resulting ${}^{\alpha}F_{nm}$, ${}^{\beta}F_{nm}$, ${}^{\alpha}H_{nm}$, and ${}^{\beta}H_{nm}$. They will be programmed as constant parameters into the subroutine for the perturbing acceleration and will be expected to serve for all subsequent runs of that subroutine, until updated. Note that the ${}^{\alpha}F_{nm}$, ${}^{\beta}F_{nm}$, ${}^{\alpha}H_{nm}$, and ${}^{\beta}H_{nm}$ *are dimensionless quantities.*

While it would be decidedly impractical to compute f^{ν}_{nm} and h^{ν}_{nm} from the following equation, for program checkout purposes it is possible to find individual, numerical values for these quantities from

$$\begin{pmatrix} f^{\nu}_{nm} \\ h^{\nu}_{nm} \end{pmatrix} = \frac{R^n}{\rho_{\nu}^{n+1}} P_n^m(\sin\theta_{\nu}) \begin{pmatrix} \cos m\lambda_{\nu} \\ \sin m\lambda_{\nu} \end{pmatrix} \tag{3-23}$$

This relationship may also be used to justify Equation 3-15, considering that

$$P_n^m(x) \equiv 0 \quad \text{for} \quad n < m \tag{3-24}$$

which is one of the basic identities of Legendre function theory.

SUBROUTINE FOR THE PERTURBING ACCELERATION

First, calculate the earth-fixed, Cartesian satellite coordinates (y_1, y_2, and y_3) from the corresponding inertial coordinates (x_1, x_2, and x_3).

$$\begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = (ABCD)_{J,n,t*} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \tag{3-51†}$$

where $(ABCD)_{J,n,t*}$ is the transformation matrix that manages the transition from the inertial frame of the satellite equations of motion (***Basic Inertial System***, which is associated with either 1950.0 or with the beginning of the day UT of trajectory epoch) to the earth-fixed reference frame corresponding to the time instant, J, n, t*. This transformation is a computer routine that is already part of TERRA/CELEST. It is external to the present algorithm. For details, see Appendices B and C of Reference 2.

Find

$$r = +\sqrt{x_1^2 + x_2^2 + x_3^2} \tag{3-52a}$$

or

$$r = +\sqrt{y_1^2 + y_2^2 + y_3^2} \tag{3-52b}$$

as convenient.

---

† Numbering sequence advanced to emphasize parallelism between certain equations of the algorithms in this and in the next chapter.

Calculate now the time dependent expansion coefficients for the tide potential from

$$F_{nm} = {}^{\alpha}F_{nm} \cos(\sigma t_i^* + \chi) + {}^{\beta}F_{nm} \sin(\sigma t_i^* + \chi) \tag{3-53}$$

$$H_{nm} = {}^{\alpha}H_{nm} \cos(\sigma t_i^* + \chi) + {}^{\beta}H_{nm} \sin(\sigma t_i^* + \chi) \tag{3-54}$$

Obtain χ as follows

$$\delta(n,m) = \begin{cases} 1 & \text{if} \quad n = m \\ 0 & \phantom{\text{if}} \quad n \neq m \end{cases} \tag{3-55}$$

$$\begin{aligned} N_{\Sigma} = {} & N\,\delta(J,\ 1975) + (365 + N)\,\delta(J,\ 1976) + (731 + N)\,\delta(J,\ 1977) \\ & + (1096 + N)\,\delta(J,\ 1978) + (1461 + N)\,\delta(J,\ 1979) \end{aligned} \tag{3-56}$$

$$\Delta T = 5.28E-04 + (3.56E-08)N_{\Sigma} \tag{3-57}$$

$$d_0 = 27392.5 + N_{\Sigma} + \Delta T \tag{3-58}$$

$$T_0 = \frac{d_0}{36525} \tag{3-59}$$

$$\chi = 270.434358 + 481267.88314137\, T_0 - .001133\, T_0^2 + .0000019\, T_0^3 \tag{3-60}$$

Note that the day number N, and thus χ, must be updated whenever t_i^* runs into the next day.

Remember that F_{nm} and H_{nm} are linear functions of two trigonometric terms. To evaluate these trigonometric terms, let t_i^* and t_{i+1}^* be the values of Universal time for which subsequent integration steps (time lines) are to be performed. Update the trigonometric time factors as follows:

$$\cos(\sigma t_{i+1}^* + \chi) = \cos\left\{(\sigma t_i^* + \chi) + \sigma\Delta t\right\} = \cos(\sigma t_i^* + \chi)\ \cos\sigma\Delta t - \sin(\sigma t_i^* + \chi)\ \sin\sigma\Delta t \tag{3-61}$$

$$\sin(\sigma t^*_{i+1} + \chi) = \sin\{(\sigma t^*_i + \chi) + \sigma\Delta t\} = \sin(\sigma t^*_i + \chi)\cos\sigma\Delta t + \cos(\sigma t^*_i + \chi)\sin\sigma\Delta t \qquad (3\text{-}62)$$

$$\Delta t = t^*_{i+1} - t^*_i \qquad (3\text{-}63)$$

Now calculate the Cartesian components of the perturbing acceleration in the earth-fixed frame

$$\frac{\partial\phi}{\partial y_1} = \sum_{n=0}^{NMAX} \sum_{m=0}^{n} \left(F_{nm}\frac{\partial U_{nm}}{\partial y_1} + H_{nm}\frac{\partial V_{nm}}{\partial y_1}\right) \qquad (3\text{-}64)$$

$$\frac{\partial\phi}{\partial y_2} = \sum_{n=0}^{NMAX} \sum_{m=0}^{n} \left(F_{nm}\frac{\partial U_{nm}}{\partial y_2} + H_{nm}\frac{\partial V_{nm}}{\partial y_2}\right) \qquad (3\text{-}65)$$

$$\frac{\partial\phi}{\partial y_3} = \sum_{n=0}^{NMAX} \sum_{m=0}^{n} \left(F_{nm}\frac{\partial U_{nm}}{\partial y_3} + H_{nm}\frac{\partial V_{nm}}{\partial y_3}\right) \qquad (3\text{-}66)$$

where

$$\frac{\partial U_{nm}}{\partial y_1} = \frac{1}{R}\left(\frac{1}{2}A_{mn}U_{n+1,m-1} - \frac{1}{2}U_{n+1,m+1}\right) \qquad (3\text{-}67)$$

$$\frac{\partial U_{nm}}{\partial y_2} = \frac{1}{R}\left(-\frac{1}{2}A_{mn}V_{n+1,m-1} - \frac{1}{2}V_{n+1,m+1}\right) \qquad (3\text{-}68)$$

$$\frac{\partial U_{nm}}{\partial y_3} = \frac{-1}{R}(n-m+1)U_{n+1,m} \qquad (3\text{-}69)$$

$$\frac{\partial V_{nm}}{\partial y_1} = \frac{1}{R}\left(\frac{1}{2}A_{mn}V_{n+1,m-1} - \frac{1}{2}V_{n+1,m+1}\right) \qquad (3\text{-}70)$$

$$\frac{\partial V_{nm}}{\partial y_2} = \frac{1}{R}\left(\frac{1}{2}A_{mn}U_{n+1,m-1} + \frac{1}{2}U_{n+1,m+1}\right) \qquad (3\text{-}71)$$

$$\frac{\partial V_{nm}}{\partial y_3} = \frac{-1}{R}(n-m+1)V_{n+1,m} \qquad (3\text{-}72)$$

and

$$A_{mn} = (n - m + 1)(n - m + 2) \tag{3-73}$$

$$U_{n,-1} = -\frac{(n-1)!}{(n+1)!}U_{n1} = -\frac{U_{n1}}{n(n+1)} \tag{3-74}$$

$$V_{n,-1} = -\frac{(n-1)!}{(n+1)!}V_{n1} = -\frac{V_{n1}}{n(n+1)} \tag{3-75}$$

Obtain the values of the individual eigenfunctions from the following recursive relationships:

$$p = \frac{R}{r} \tag{3-76}$$

$$\sin\psi = \frac{y_3}{r} \tag{3-77}$$

$$V_{no} = 0 \quad \text{for all values of n} \tag{3-78}$$

$$U_{nm} = V_{nm} = 0 \quad \text{for all } n < m \tag{3-79}$$

To advance in n, evaluate

$$U_{n+1,m} = \frac{p}{n-m+1}\left\{(2n+1)\sin\psi\, U_{nm} - (n+m)p\, U_{n-1,m}\right\} \tag{3-80}$$

$$V_{n+1,m} = \frac{p}{n-m+1}\left\{(2n+1)\sin\psi\, V_{nm} - (n+m)p\, V_{n-1,m}\right\} \tag{3-81}$$

To advance in m, use

$$U_{n+1,n+1} = (2n+1)p\left(\frac{y_1}{r}U_{nn} - \frac{y_2}{r}V_{nn}\right) \tag{3-82}$$

$$V_{n+1,n+1} = (2n+1)p\left(\frac{y_1}{r} V_{nn} + \frac{y_2}{r} U_{nn}\right) \tag{3-83}$$

Start from

$$U_{0,0} = \frac{\mu_E}{r} \tag{3-84}$$

$$U_{1,0} = \mu_E \frac{R\, y_3}{r^3} \tag{3-85}$$

$$V_{0,0} = V_{1,0} = 0 \tag{3-86}$$

The Cartesian components of the perturbing acceleration, in the earth-fixed reference frame, are now

$$T_{y_1} = \frac{\partial \phi}{\partial y_1} \tag{3-87}$$

$$T_{y_2} = \frac{\partial \phi}{\partial y_2} \tag{3-88}$$

$$T_{y_3} = \frac{\partial \phi}{\partial y_3} \tag{3-89}$$

Finally, rotate the perturbing acceleration back into the inertial coordinate system

$$\begin{pmatrix} T_{x_1} \\ T_{x_2} \\ T_{x_3} \end{pmatrix} = (ABCD)^T_{J,n,t*} \begin{pmatrix} T_{y_1} \\ T_{y_2} \\ T_{y_3} \end{pmatrix} \tag{3-90}$$

where $(ABCD)^T$ is the transpose of (ABCD).

COMPUTER PROGRAM OUTPUT

The inertial components (T_{x_1}, T_{x_2}, and T_{x_3}) of the perturbing acceleration due to the presence of the ocean tide are output in terms of km sec^{-2}. If required, convert to m sec^{-2}.

TRIAL DATA FOR PROGRAM CHECKOUT

Non-zero tidal heights and associated phase values were assumed for nine surface area elements. Several of the ${}^{\alpha}F_{nm}$, ${}^{\beta}F_{nm}$, ${}^{\alpha}H_{nm}$, and ${}^{\beta}H_{nm}$ were subsequently computed. The algorithms for the perturbing acceleration, featured in the first and second computer routines for the ocean tide, are identical. Trial data for these segments of the two tide routines will therefore be listed only once, namely, for the second ocean tide routine, below.

Quantities Associated With the Surface Area Elements

i,j	ν	ζ m	δ deg	θ rad	λ rad	ρ km	ΔS_ν km^2	α_ν $km^3\ sec^{-2}$	β_ν $km^3\ sec^{-2}$
1,1	1	10	25	1.562069681	8.7266463 E − 03	6356.800824	108.1411251	6.5403462E − 08	3.0498135E − 08
2,1	2	10	25	1.562069681	2.61799388E − 02	6356.800824	108.1411251	6.5403462E − 08	3.0498135E − 08
3,1	3	10	25	1.562069681	4.36332313E − 02	6356.800824	108.1411251	6.5403462E − 08	3.0498135E − 08
1,2	4	10	25	1.544616388	8.7266463 E − 03	6356.813825	432.4766612	2.6156072E − 07	1.2196777E − 07
2,2	5	20	30	1.544616388	2.61799388E − 02	6356.813825	432.4766612	4.9987043E − 07	2.8860033E − 07
3,2	6	20	30	1.544616388	4.36332312E − 02	6356.813825	432.4766612	4.9987043E − 07	2.8860033E − 07
1,3	7	10	25	1.527163095	8.7266463 E − 03	6356.839812	648.550316	3.9224149E − 07	1.8290521E − 07
2,3	8	20	30	1.527163095	2.61799388E − 02	6356.839812	648.550316	7.496153 E − 07	4.327906 E − 07
3,3	9	20	30	1.527163095	4.36332313E − 02	6356.839812	648.550316	7.496153 E − 07	4.327906 E − 07

Values for the f_{nm}^{ν} and h_{nm}^{ν}

ν	$f_{0,0}^{\nu}$ km^{-1}	$h_{0,0}^{\nu}$ km^{-1}	$f_{1,0}^{\nu}$ km^{-1}	$h_{1,0}^{\nu}$ km^{-1}
1	1.5731184E − 04	0	1.5783403E − 04	0
2	1.5731184E − 04	0	1.5783403E − 04	0
3	1.5731184E − 04	0	1.5783403E − 04	0
4	1.5731151E − 04	0	1.5778531E − 04	0
5	1.5731151E − 04	0	1.5778531E − 04	0
6	1.5731151E − 04	0	1.5778531E − 04	0
7	1.5731087E − 04	0	1.5768788E − 04	0
8	1.5731087E − 04	0	1.5768788E − 04	0
9	1.5731087E − 04	0	1.5768788E − 04	0

ν	$f_{1,1}^{\nu}$ km^{-1}	$h_{1,1}^{\nu}$ km^{-1}	$f_{2,0}^{\nu}$ km^{-1}	$h_{2,0}^{\nu}$ km^{-1}
1	1.3773442E − 06	1.2019901E − 08	1.5835193E − 04	0
2	1.3769246E − 06	3.6056041E − 08	1.5835193E − 04	0
3	1.3760857E − 06	6.0081198E − 08	1.5835193E − 04	0
4	4.1315963E − 06	3.6055895E − 08	1.5820627E − 04	0
5	4.1303378E − 06	1.0815670E − 07	1.5820627E − 04	0
6	4.1278211E − 06	1.8022456E − 07	1.5820627E − 04	0
7	6.8845393E − 06	6.0080464E − 08	1.5791513E − 04	0
8	6.8824422E − 06	1.8022309E − 07	1.5791513E − 04	0
9	6.8782468E − 06	3.0031082E − 07	1.5791513E − 04	0

ν	$f_{2,1}^{\nu}$ km^{-1}	$h_{2,1}^{\nu}$ km^{-1}	$f_{2,2}^{\nu}$ km^{-1}	$h_{2,2}^{\nu}$ km^{-1}
1	4.1457488E − 06	3.6179402E − 08	3.6175267E − 08	6.3144163E − 10
2	4.1444859E − 06	1.0852718E − 07	3.6131193E − 08	1.8935556E − 09
3	4.1419607E − 06	1.8084191E − 07	3.6043099E − 08	3.1533625E − 09
4	1.2432120E − 05	1.0849347E − 07	3.2550933E − 07	5.6817864E − 09
5	1.2428333E − 05	3.2544735E − 07	3.2511274E − 07	1.7038437E − 08
6	1.2420760E − 05	5.4230210E − 07	3.2432006E − 07	2.8374329E − 08
7	2.0703116E − 05	1.8067335E − 07	9.0381431E − 07	1.5776137E − 08
8	2.0696809E − 05	5.4196503E − 07	9.0271315E − 07	4.7309191E − 08
9	2.0684198E − 05	9.0309161E − 07	9.0051217E − 07	7.8784606E − 08

Values for the f^{ν}_{nm} and h^{ν}_{nm} (Continued)

ν	$f^{\nu}_{3,0}$ km^{-1}	$h^{\nu}_{3,0}$ km^{-1}	$f^{\nu}_{3,1}$ km^{-1}	$h^{\nu}_{3,1}$ km^{-1}
1	1.5886548E − 04	0	8.3188628E − 06	7.2597615E − 08
2	1.5886548E − 04	0	8.3163287E − 06	2.1777073E − 07
3	1.5886548E − 04	0	8.3112614E − 06	3.6287751E − 07
4	1.5857391E − 04	0	2.4934851E − 05	2.1760315E − 07
5	1.5857391E − 04	0	2.4927255E − 05	6.5274316E − 07
6	1.5857391E − 04	0	2.4912067E − 05	1.0876843E − 06
7	1.5799154E − 04	0	4.1485684E − 05	3.6204008E − 07
8	1.5799154E − 04	0	4.1473047E − 05	1.0860100E − 06
9	1.5799154E − 04	0	4.1447777E − 05	1.8096490E − 06

ν	$f^{\nu}_{3,2}$ km^{-1}	$h^{\nu}_{3,2}$ km^{-1}	$f^{\nu}_{3,3}$ km^{-1}	$h^{\nu}_{3,3}$ km^{-1}
1	1.8147675E − 07	3.1676885E − 09	1.5834220E − 09	4.1463365E − 11
2	1.8125565E − 07	9.4992060E − 09	1.579082 E − 09	1.2427645E − 10
3	1.8081372E − 07	1.5819151E − 08	1.5704138E − 09	2.0674889E − 10
4	1.6324485E − 06	2.8494495E − 08	4.2739029E − 08	1.1191609E − 09
5	1.6304596E − 06	8.5448767E − 08	4.2621885E − 08	3.3544151E − 09
6	1.6264843E − 06	1.4229893E − 07	4.2387916E − 08	5.58047 E − 09
7	4.5299018E − 06	7.9069730E − 08	1.9774213E − 07	5.1780599E − 09
8	4.5243828E − 06	2.3711286E − 07	1.9720013E − 07	1.5519987E − 08
9	4.5133516E − 06	3.9486710E − 07	1.9611762E − 07	2.5819375E − 08

ν	$f^{\nu}_{4,3}$ km^{-1}	$h^{\nu}_{4,3}$ km^{-1}
1	1.1120747E − 08	2.9120701E − 10
2	1.1090266E − 08	8.7282284E − 10
3	1.1029387E − 08	1.4520463E − 09
4	3.0007426E − 07	7.8577211E − 09
5	2.9925178E − 07	2.3551626E − 08
6	2.9760907E − 07	3.9180977E − 08
7	1.3875122E − 06	3.6333286E − 08
8	1.3837091E − 06	1.0890027E − 07
9	1.3761134E − 06	1.8116877E − 07

Time-Independent Coefficients of the Tide Potential

n	m	${}^{\alpha}F_{nm}$	${}^{\beta}F_{nm}$	${}^{\alpha}H_{nm}$	${}^{\beta}H_{nm}$
0	0	8.4018456E − 12	4.6140106E − 12	0	0
1	0	8.3681641E − 12	4.5954868E − 12	0	0
1	1	2.9297877E − 13	1.6202500E − 13	4.3123569E − 15	2.4544968E − 15
2	0	8.3290386E − 12	4.5739464E − 12	0	0
2	1	2.9177584E − 13	1.6135948E − 13	4.2946458E − 15	2.4444118E − 15
2	2	2.7840515E − 15	1.5423222E − 15	8.2132887E − 17	4.683368 E − 17
3	0	8.2845357E − 12	4.5494263E − 12	0	0
3	1	2.9046632E − 13	1.6063487E − 13	4.2753632E − 15	2.4334303E − 15
3	2	2.7724443E − 15	1.5358913E − 15	8.1790436E − 17	4.6638388E − 17
3	3	1.5813081E − 17	1.0259185E − 17	8.2068535E − 19	4.6816230E − 19
4	3	1.8435105E − 17	1.0215973E − 17	8.1722863E − 19	4.6619036E − 19

4 – SECOND COMPUTER ROUTINE FOR THE PERTURBING ACCELERATION CAUSED BY THE M_2 COMPONENT OF THE OCEAN TIDE (BASED ON A FUNCTIONALIZATION OF THE SEA SURFACE)

INPUT DATA

$\left.\begin{matrix} C_{ij} \\ S_{ij} \\ C'_{ij} \\ S'_{ij} \end{matrix}\right\}$ NOAA/Goad expansion coefficients of the tidal surface, in meters; these are specified by a computer card deck

R = ***radius of the earth*** (semimajor axis of a suitable reference ellipsoid) in kilometers
suggested value: R = 6378.145 km

G = Newton's constant, in kilometers, kilograms, and seconds
suggested value: G = 6.6732E − 20 $km^3\ kg^{-1}\ sec^{-2}$

μ_E = gravitational constant of the earth, in kilometers and seconds
suggested value: μ_E = 398601 $km^3\ sec^{-2}$

ρ = density of water, in kilograms and kilometers
suggested value: ρ = 1.E + 12 kg km^{-3}

σ = rate of mean mean longitude of moon

$$\text{suggested value: } \sigma = \frac{180}{\pi} 1.40519\text{E} - 04 \text{ deg sec}^{-1}$$

NMAX = limit on the number of terms in the expansion of the tide potential; NMAX is equal to the largest value of the index i for the expansion coefficients of the tidal surface

The parameters J, n, and t* specify the time instant for which the perturbing acceleration due to the tide is to be found. Normally, this is the time instant associated with the current ***time line*** of the orbit integration.

J = number of the calendar year
n = number of the day within the year
t* = mean solar time at Greenwich (GMT, UTC), in seconds

$\left.\begin{matrix} x_1 \\ x_2 \\ x_3 \end{matrix}\right\}$ = inertial, Cartesian components of satellite position vector, in kilometers, associated with the just specified time instant (for a precise definition of the reference frame, see Appendices B and C of Reference 2)

SUBROUTINE FOR THE TIDE POTENTIAL COEFFICIENTS

This is the first part of the algorithm for the second ocean tide computer routine. The time-independent constituents of the expansion coefficients of the ocean tide potential (F'_{ij}, F''_{ij}, H'_{ij}, H''_{ij}) are calculated from the expansion coefficients for the NOAA/Goad tidal ocean surface. Once established, the latter remain unchanged as long as the current version of the NSWC/Schwiderski Ocean Tide Model (on which the NOAA/Goad tidal surface is based) remains valid. Thus, the subroutine under discussion will be exercised quite infrequently, namely, just once each time a new edition of the NSWC/Schwiderski Ocean Tide Model becomes available. That is expected to occur at intervals of several years, during which the present subroutine will remain dormant.

$$F'_{ij} = \frac{2\pi R^2 G\rho}{\mu_E} \frac{2}{2i+1} 10^{-3} C_{ij} \tag{4-1}$$

$$F''_{ij} = \frac{2\pi R^2 G\rho}{\mu_E} \frac{2}{2i+1} 10^{-3} C'_{ij} \tag{4-2}$$

$$H'_{ij} = \frac{2\pi R^2 G\rho}{\mu_E} \frac{2}{2i+1} 10^{-3} S_{ij} \tag{4-3}$$

$$H''_{ij} = \frac{2\pi R^2 G\rho}{\mu_E} \frac{2}{2i+1} 10^{-3} S'_{ij} \tag{4-4}$$

SUBROUTINE FOR THE PERTURBING ACCELERATION

Proceed exactly as in Equations 3-51 through 3-90 from the first ocean tide computer routine, but delete Equations 3-53 and 3-54 and replace them with

$$F_{nm} = F'_{nm} \cos(\sigma t_i^* + \chi) + F''_{nm} \sin(\sigma t_i^* + \chi) \tag{4-53}$$

$$H_{nm} = H'_{nm} \cos(\sigma t_i^* + \chi) + H''_{nm} \sin(\sigma t_i^* + \chi) \tag{4-54}$$

COMPUTER PROGRAM OUTPUT

The inertial components (T_{x_1}, T_{x_2}, and T_{x_3}) of the perturbing acceleration due to the presence of the ocean tide are output in terms of km sec^{-2}. If required, convert to m sec^{-2}.

TRIAL DATA FOR PROGRAM CHECKOUT

Non-zero values were assigned to selected NOAA/Goad expansion coefficients, and the corresponding expansion coefficients of the tide potential were computed. The algorithm for the perturbing acceleration was then exercised with the following results.

J = 1977		
n = 202		
t^* = 50000	sec
R = 6378.145	km
G = 6.6732E - 20	$km^3\ kg^{-1}\ sec^{-2}$
μ_E = 398601	$km^3\ sec^{-2}$

$$\rho = 10^{12} \qquad \text{kg km}^{-3}$$

$$\sigma = \frac{180}{\pi} 1.40519\text{E} - 04 \qquad \text{deg sec}^{-1}$$

$$\text{NMAX} = 4$$

$$C_{20} = +0.2906060089\text{E} - 01 \text{ m}$$

$$C_{40} = -0.107121752\text{E} + 00 \text{ m}$$

$$C_{43} = +0.435761219\text{E} - 04 \text{ m}$$

$$C'_{20} = -0.4424413130\text{E} - 01 \text{ m}$$

$$C'_{40} = +0.873468034\text{E} - 01 \text{ m}$$

$$C'_{43} = -0.160563906\text{E} - 02 \text{ m}$$

$$S_{20} = 0$$

$$S_{40} = 0$$

$$S_{43} = -0.363303008\text{E} - 02 \text{ m}$$

$$S'_{20} = 0$$

$$S'_{40} = 0$$

$$S'_{43} = -0.264356490\text{E} - 02 \text{ m}$$

$$F'_{20} = +4.9742658\text{E} - 10$$

$$F'_{40} = -1.0186607\text{E} - 09$$

$$F'_{43} = +4.1438160\text{E} - 13$$

$F''_{20} = -7.57321111E-10$

$F''_{40} = +8.30613340E-10$

$F''_{43} = -1.5268621E-11$

$H'_{20} = 0$

$H'_{40} = 0$

$H'_{43} = -3.4547839E-11$

$H''_{20} = 0$

$H''_{40} = 0$

$H''_{43} = -2.5138645E-11$

$N_{\Sigma} = 933$

$\Delta T = 0.000561$

$d_0 = 28325.50056$

$T_0 = 0.7755099401$

$\chi = 373498.4609$

$\sigma = 0.0080511456$ deg/sec

$\sigma t^* = 402.557282$ deg

$\cos(\sigma t^* + \chi) = -0.754501$

$\sin(\sigma t^* + \chi) = -0.656298$

$F_{20} = +1.2171968E-10$

$H_{20} = 0$

$F_{40} = +2.2345060E-10$

$H_{40} = 0$

$F_{43} = +9.7081216E-12$

$H_{43} = +4.2564846E-11$

$x_1 = 3151.52923$ km

$x_2 = 5458.60875$ km

$x_3 = 3639.07250$ km

From external source

$$(ABCD) = \begin{pmatrix} -0.8405285753 & 0.5417623775 & 0.2289080162E-02 \\ -0.5417605355 & -0.8405316908 & 0.1413662999E-02 \\ 0.2689913850E-02 & -0.5190827376E-04 & 0.9999963803 \end{pmatrix}$$

$y_1 = +316.64861$ km

$y_2 = -6290.36338$ km

$y_3 = +3647.25332$ km

$\frac{\partial U_{20}}{\partial y_1} = -0.0000964047$ km/sec^2

$\frac{\partial V_{20}}{\partial y_1} = -4.7E-13$ km/sec^2

$\frac{\partial U_{20}}{\partial y_2} = +0.0019151219$ km/sec^2

$\frac{\partial V_{20}}{\partial y_2} = -2.4E-14$ km/sec^2

Values for the Eigenfunctions of the Tide Potential, in $km^2\ sec^{-2}$

n	m	U_{nm}	V_{nm}
0	−1	0	0
0	0	54.76683963	0
1	−1	−1.044042677	−20.74036524
1	0	24.05119113	0
1	1	2.088085354	−41.48073047
2	−1	−0.4584976993	−9.108257692
2	0	−5.186455141	0
2	1	2.750986196	−54.64954615
2	2	−94.01442066	−9.489169689
3	−1	−0.0512402699	−1.017910413
3	0	−16.10992273	0
3	1	0.6148832385	−12.21492495
3	2	−206.435027	−20.83613333
3	3	−53.85812191	354.226451
4	−1	0.109342534	2.172137348
4	0	−9.393545781	0
4	1	−2.18685068	43.44274695
4	2	−136.7982658	−13.80747707
4	3	−165.56486	1088.924921
4	4	1863.67849	380.085929
5	−1	0.0917032956	1.821726148
5	0	2.472201758	0
5	1	−2.751098868	54.65178443
5	2	136.846714	13.8123671
5	3	−182.4236505	1199.805712
5	4	7366.011828	1502.253452
6	−1	0.0153004451	0.303950046
6	0	8.002095405	0
6	1	−0.6426186943	12.76590193
6	2	349.1179421	35.23756643
6	3	45.32033435	−298.0731712
6	4	11350.89177	2314.94556

$$\frac{\partial U_{20}}{\partial y_3} = +0.0075774019 \quad \text{km/sec}^2$$

$$\frac{\partial V_{20}}{\partial y_3} = 0$$

$$\frac{\partial U_{40}}{\partial y_1} = +0.0004313321 \quad \text{km/sec}^2$$

$$\frac{\partial V_{40}}{\partial y_1} = +7.8E-13 \quad \text{km/sec}^2$$

$$\frac{\partial U_{40}}{\partial y_2} = -0.0085686018 \quad \text{km/sec}^2$$

$$\frac{\partial V_{40}}{\partial y_2} = 0$$

$$\frac{\partial U_{40}}{\partial y_3} = -0.0019380257 \quad \text{km/sec}^2$$

$$\frac{\partial V_{40}}{\partial y_3} = 0$$

$$\frac{\partial U_{43}}{\partial y_1} = -0.5130748473 \quad \text{km/sec}^2$$

$$\frac{\partial V_{43}}{\partial y_1} = -0.1112689700 \quad \text{km/sec}^2$$

$$\frac{\partial U_{43}}{\partial y_2} = -0.1242624348 \quad \text{km/sec}^2$$

$$\frac{\partial V_{43}}{\partial y_2} = +0.6418082461 \quad \text{km/sec}^2$$

$$\frac{\partial U_{43}}{\partial y_3} = +0.0572027292 \qquad \text{km/sec}^2$$

$$\frac{\partial V_{43}}{\partial y_3} = -0.3762240313 \qquad \text{km/sec}^2$$

$$T_{y_1} = -9.632495E-12 \qquad \text{km/sec}^2$$

$$T_{y_2} = +2.443056E-11 \qquad \text{km/sec}^2$$

$$T_{y_3} = -1.4969321E-11 \qquad \text{km/sec}^2$$

$$T_{x_1} = -5.179392E-12 \qquad \text{km/sec}^2$$

$$T_{x_2} = -2.5752406E-11 \qquad \text{km/sec}^2$$

$$T_{x_3} = -1.495676E-11 \qquad \text{km/sec}^2$$

5 – COMPUTER ROUTINE FOR THE PERTURBING ACCELERATION CAUSED BY THE TIDES OF THE SOLID EARTH

INPUT DATA

The parameters J, n, and t* specify the time instant t for which the perturbing acceleration due to the tide is to be found. Normally, this is the time instant associated with the current ***time line*** of the orbit integration.

J = number of the calendar year
n = number of the day within the year
t* = mean solar time at Greenwich (GMT, UTC), in seconds

Δt = tidal lag parameter, in seconds

$\tilde{\omega}$ = earth's sidereal rate of rotation
suggested value: $\tilde{\omega} = 4.178074622E-03$ deg sec^{-1}

$$\left.\begin{matrix} x_L'' \\ y_L'' \\ z_L'' \end{matrix}\right\} =$$ the inertial, Cartesian coordinates of the moon, in kilometers, for the time instant $(t - \Delta t)$, from the TERRA/CELEST moon ephemeris; the reference frame is referred to mean equator and mean equinox at time instant t_1

$$\left.\begin{matrix} x_S'' \\ y_S'' \\ z_S'' \end{matrix}\right\} =$$ the inertial, Cartesian coordinates of the sun, in kilometers, for the time instant $(t - \Delta t)$, from the TERRA/CELEST sun ephemeris; the reference frame is referred to mean equator and mean equinox at time instant t_1

t_1 = epoch time for the moon ephemeris and sun ephemeris, in Julian Date
t_1 = JD2433282.423

$$\left.\begin{matrix} x_1 \\ x_2 \\ x_3 \end{matrix}\right\} =$$ inertial, Cartesian components of satellite position vector, in kilometers, associated with the time instant $t(J, n, t^*)$ (for a precise definition of the reference frame, see Appendices B and C of Reference 2)

$$\left.\begin{matrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{matrix}\right\} =$$ Cartesian satellite velocity components associated with time instant t, in km sec^{-1}

μ_E = gravitational constant of the earth, in kilometers and seconds
suggested value: $\mu_E = 398601 \text{ km}^3 \text{ sec}^{-2}$

R = ***radius of the earth*** (semimajor axis of a suitable reference ellipsoid) in kilometers
suggested value: R = 6378.145 km

ϵ^2 = square of eccentricity of reference ellipsoid
suggested value: $\epsilon^2 = .006693421623$
in case it is desired to start from the flattening, f, find ϵ^2 from $\epsilon^2 = (2 - f)f$

k_{ij} = expansion coefficients for the Love numbers
suggested values: $k_{20} = 0.3$
$k_{21} = 0.01$
$k_{30} = 0.1$
$k_{31} = 0.01$
$k_{22} = 0.1$

m' = mass of moon or sun, in kilograms

suggested values: m'_L = mass of moon = 7.3693281E + 22 kg

m'_S = mass of sun = 1.99E + 30 kg

M = mass of earth, in kilograms

suggested value: M = 5.9731613E + 24 kg

a' = semimajor axis of moon or sun orbit relative to earth, in kilometers; select values that are compatible with the moon ephemeris or sun ephemeris (optionally, values may be obtained from a current edition of the ***AMERICAN EPHEMERIS***)

suggested values: a'_L = 384400 km

a'_S = 149500000 km

SUBROUTINE FOR THE POSITION OF THE FICTITIOUS MOON

$$\begin{pmatrix} x'_L \\ y'_L \\ z'_L \end{pmatrix} = \begin{pmatrix} X_x & Y_x & Z_x \\ X_y & Y_y & Z_y \\ X_z & Y_z & Z_z \end{pmatrix} \begin{pmatrix} x''_L \\ y''_L \\ z''_L \end{pmatrix} \quad (5\text{-}1)$$

$$X_x = \cos \xi_0 \cos \theta \cos z - \sin \xi_0 \sin z \quad (5\text{-}2)$$

$$Y_x = - \sin \xi_0 \cos \theta \cos z - \cos \xi_0 \sin z \quad (5\text{-}3)$$

$$Z_x = - \sin \theta \cos z \quad (5\text{-}4)$$

$$X_y = \cos \xi_0 \cos \theta \sin z + \sin \xi_0 \cos z \quad (5\text{-}5)$$

$$Y_y = - \sin \xi_0 \cos \theta \sin z + \cos \xi_0 \cos z \quad (5\text{-}6)$$

$$Z_y = - \sin \theta \sin z \quad (5\text{-}7)$$

$$X_z = \cos \xi_0 \sin \theta \quad (5\text{-}8)$$

$$Y_z = - \sin \xi_0 \sin \theta \quad (5\text{-}9)$$

$$Z_z = \cos \theta \quad (5\text{-}10)$$

$$\xi_0 = \left\{(2304.250 + 1.396T_0)T + 0.302T^2 + 0.018T^3\right\} \frac{1}{3600} \qquad (5\text{-}11)$$

$$z = \xi_0 + \left\{0.791T^2\right\} \frac{1}{3600} \qquad (5\text{-}12)$$

$$\theta = \left\{(2004.682 - 0.853T_0)T - 0.426T^2 - 0.042T^3\right\} \frac{1}{3600} \qquad (5\text{-}13)$$

ξ_0, z, and θ result in degrees.

$$T_0 = (t_1 - 2415020.313)(0.273790926497E - 04) \qquad (5\text{-}14)$$

$$T = (0.273790926497E - 04)(t_2 - t_1) \qquad (5\text{-}15)$$

$$(t_2 - t_1) = \left\{(J - 50)365 + \Delta_0 + (n - 1) + .077\right\} \qquad (5\text{-}16)$$

$$\Delta_0 = \mathrm{INT}\left(\frac{J - 53}{4} + 1\right) \qquad (5\text{-}17)$$

The Cartesian coordinates of the fictitious moon (x_L^*, y_L^*, and z_L^*) referred to the coordinate system in which the satellite orbit integration is performed may now be computed:

$$\begin{pmatrix} x_L^* \\ y_L^* \\ z_L^* \end{pmatrix} = \begin{pmatrix} \cos\tilde{\omega}\Delta t & -\sin\tilde{\omega}\Delta t & 0 \\ \sin\tilde{\omega}\Delta t & \cos\tilde{\omega}\Delta t & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_L' \\ y_L' \\ z_L' \end{pmatrix} \qquad (5\text{-}18)$$

Also, compute

$$r_L^* = +\sqrt{(x_L^*)^2 + (y_L^*)^2 + (z_L^*)^2} \qquad (5\text{-}19)$$

$$\lambda_L^* = \frac{x_L^*}{r_L^*} \qquad (5\text{-}20)$$

$$\mu_L^* = \frac{y_L^*}{r_L^*} \qquad (5\text{-}21)$$

$$V_L^* = \frac{z_L^*}{r_L^*} \qquad (5\text{-}22)$$

SUBROUTINE FOR THE POSITION OF THE FICTITIOUS SUN

$$\begin{pmatrix} x'_S \\ y'_S \\ z'_S \end{pmatrix} = \begin{pmatrix} X_x & Y_x & Z_x \\ X_y & Y_y & Z_y \\ X_z & Y_z & Z_z \end{pmatrix} \begin{pmatrix} x''_S \\ y''_S \\ z''_S \end{pmatrix} \tag{5-23}$$

Evaluate the matrix elements X_x, Y_x, etc., according to Equations 5-2 through 5-17. Then, calculate the Cartesian coordinates of the fictitious sun (x^*_S, y^*_S, z^*_S) referred to the coordinate system in which the satellite orbit integration is performed:

$$\begin{pmatrix} x^*_S \\ y^*_S \\ z^*_S \end{pmatrix} = \begin{pmatrix} \cos \tilde{\omega}\Delta t & -\sin \tilde{\omega}\Delta t & 0 \\ \sin \tilde{\omega}\Delta t & \cos\tilde{\omega}\Delta t & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x'_S \\ y'_S \\ z'_S \end{pmatrix} \tag{5-24}$$

Further, compute

$$r^*_S = +\sqrt{(x^*_S)^2 + (y^*_S)^2 + (z^*_S)^2} \tag{5-25}$$

$$\lambda^*_S = \frac{x^*_S}{r^*_S} \tag{5-26}$$

$$\mu^*_S = \frac{y^*_S}{r^*_S} \tag{5-27}$$

$$\nu^*_S = \frac{z^*_S}{r^*_S} \tag{5-28}$$

SUBROUTINE FOR SATELLITE POSITION

$$r = +\sqrt{x_1^2 + x_2^2 + x_2^2} \tag{5-29}$$

$$\lambda = \frac{x_1}{r} \tag{5-30}$$

$$\mu = \frac{x_2}{r} \tag{5-31}$$

$$\nu = \frac{x_3}{r} \tag{5-32}$$

$$v^2 = (\dot{x}_1)^2 + (\dot{x}_2)^2 + (\dot{x}_3)^2 \tag{5-33}$$

$$a = \left(\frac{2}{r} - \frac{v^2}{\mu_E}\right)^{-1} \tag{5-34}$$

SUBROUTINE FOR THE PERTURBING ACCELERATION

T_{x_1}, T_{x_2}, and T_{x_3} are the Cartesian components of the perturbing acceleration, in km sec^{-2}. These are referred to the (inertial) reference frame in which the satellite orbit integration is performed, and are thus readily suited for insertion into the TERRA/CELEST equations of motion.

$$T_{x_1} = \frac{-1}{r^3}\left\{\left[C_0 + \frac{3C_1V_1}{r} + \frac{C_2}{r^2}(5V_2 - 2P_0') + \frac{C_3}{r^3}(7V_3 - F) + \frac{C_4}{r^4}(9V_4 - H)\right]x_1 - \left[\frac{C_1}{5}A_3' + \frac{C_2}{r}\rho_{11} + \frac{C_3}{r^2}\rho_{12} + \frac{C_4}{r^3}\rho_{13}\right]\right\} \tag{5-35}$$

$$T_{x_2} = \frac{-1}{r^3}\left\{\left[C_0 + \frac{3C_1V_1}{r} + \frac{C_2}{r^2}(5V_2 - 2P_0') + \frac{C_3}{r^3}(7V_3 - F) + \frac{C_4}{r^4}(9V_4 - H)\right]x_2 - \left[\frac{C_1}{5}A_4' + \frac{C_2}{r}\rho_{21} + \frac{C_3}{r^2}\rho_{22} + \frac{C_4}{r^3}\rho_{23}\right]\right\} \tag{5-36}$$

$$T_{x_3} = \frac{-1}{r^3}\left\{\left[C_0 + \frac{3C_1V_1}{r} + \frac{C_2}{r^2}(5V_2 - 2P_0') + \frac{C_3}{r^3}(7V_3 - F) + \frac{C_4}{r^4}(9V_4 - H)\ x_3\right] - \left[\frac{-4C_1}{5}A_0' + \frac{C_2}{r}\rho_{31} + \frac{C_3}{r^2}\rho_{32} + \frac{C_4}{r^3}\rho_{33}\right]\right\} \tag{5-37}$$

$$\rho_{11} = 2\lambda P_1' + \mu P_2' + \nu P_3' \tag{5-38}$$

$$\rho_{12} = (1 - 5\nu^2)S_1' + 3(\lambda^2 - \mu^2)S_4' + 2\lambda(3\mu S_5' + \nu S_6') + \mu\nu S_7' \tag{5-39}$$

$$\rho_{13} = \nu\left(1 - \frac{7}{3}\nu^2\right)T_1' + 3\nu(\lambda^2 - \mu^2)T_4' + 6\lambda\mu\nu T_5' + (1 - 7\nu^2)(2\lambda T_6' + \mu T_7') \tag{5-40}$$

$$\rho_{21} = -2\mu P_1' + \lambda P_2' + \nu P_4' \tag{5-41}$$

$$\rho_{22} = (1 - 5\nu^2)S_2' - 6\lambda\mu S_4' + 3(\lambda^2 - \mu^2)S_5' + \nu(\lambda S_7' - 2\mu S_6') \tag{5-42}$$

$$\rho_{23} = \nu\left(1 - \frac{7}{3}\nu^2\right)T_2' - 6\lambda\mu\nu T_4' + 3\nu(\lambda^2 - \mu^2)T_5' + (1 - 7\nu^2)(\lambda T_7' - 2\mu T_6') \tag{5-43}$$

$$\rho_{31} = -6\nu P_0' + \lambda P_3' + \mu P_4' \tag{5-44}$$

$$\rho_{32} = -10\nu(\lambda S_1' + \mu S_2') + (3 - 15\nu^2)S_3' + (\lambda^2 - \mu^2)S_6' + \lambda\mu S_7' \tag{5-45}$$

$$\begin{aligned}\rho_{33} = {} & (1 - 7\nu^2)(\lambda T_1' + \mu T_2') - 20\nu(3 - 7\nu^2)T_3' \\ & + \lambda(\lambda^2 - 3\mu^2)T_4' + \mu(3\lambda^2 - \mu^2)T_5' \\ & - 14\nu[(\lambda^2 - \mu^2)T_6' + \lambda\mu T_7']\end{aligned} \tag{5-46}$$

$$F = 2(\lambda S_1' + \mu S_2' + 3\nu S_3') \tag{5-47}$$

$$H = 2\left[\nu(\lambda T_1' + \mu T_2') + 6(1 - 5\nu^2)T_3' + (\lambda^2 - \mu^2)T_6' + \lambda\mu T_7'\right] \tag{5-48}$$

where

$$C_0 = \left(\frac{2}{3}\epsilon^2 k_{20} - \frac{2}{5}k_{22}\right)aKA_0' \tag{5-49}$$

$$C_1 = K\alpha k_{21} a^2 \tag{5-50}$$

$$C_2 = K\alpha^2 a^3 \tag{5-51}$$

$$C_3 = K\alpha^3 a^4 \tag{5-52}$$

$$C_4 = K\alpha^4 a^5 \tag{5-53}$$

and

$$K = \frac{m'}{M} n^2 a^2 \alpha'^3 \beta^{*3} \tag{5-54}$$

$$a^3 n^2 = \mu = MG \tag{5-55}$$

$$\alpha' = \frac{R}{a'} \tag{5-56}$$

$$\alpha = \frac{R}{a} \tag{5-57}$$

$$\beta^* = \frac{a'}{r^*} \tag{5-58}$$

and

$$V_0 \equiv 1 \tag{5-59}$$

$$V_1 = \frac{1}{5}(A'_3 \lambda + A'_4 \mu - 4A'_0 \nu) \tag{5-60}$$

$$V_2 = \sum_{n=0}^{4} P'_n P_n \tag{5-61}$$

$$V_3 = \sum_{n=1}^{7} S'_n S_n \tag{5-62}$$

$$V_4 = \sum_{n=1}^{7} T'_n T_n \tag{5-63}$$

and

$$A'_0 = \frac{1}{4}(1 - 3\nu^{*2}) \tag{5-64}$$

$$A'_1 = \frac{3}{4}(\lambda^{*2} - \mu^{*2}) \tag{5-65}$$

$$A'_2 = 3\lambda^* \mu^* \tag{5-66}$$

$$A'_3 = 3\lambda^* \nu^* \tag{5-67}$$

$$A'_4 = 3\mu^* \nu^* \tag{5-68}$$

and

$$B_1' = \frac{3}{8}\lambda^*(1 - 5\nu^{*2}) \tag{5-69}$$

$$B_2' = \frac{3}{8}\mu^*(1 - 5\nu^{*2}) \tag{5-70}$$

$$B_3' = \frac{1}{4}\nu^*(3 - 5\nu^{*2}) \tag{5-71}$$

$$B_4' = \frac{5}{8}\lambda^*(\lambda^{*2} - 3\mu^{*2}) \tag{5-72}$$

$$B_5' = \frac{5}{8}\mu^*(3\lambda^{*2} - \mu^{*2}) \tag{5-73}$$

$$B_6' = \frac{15}{4}\nu^*(\lambda^{*2} - \mu^{*2}) \tag{5-74}$$

$$B_7' = 15\lambda^*\mu^*\nu^* \tag{5-75}$$

and

$$P_0' = \left(k_{20} + \frac{2}{7}k_{22}\right)\left(1 - \frac{55}{42}\epsilon^2\right)A_0' + \frac{3}{7}k_{31}\alpha'\beta^*B_3' \tag{5-76}$$

$$P_0 = 1 - 3\nu^2 \tag{5-77}$$

$$P_1' = \left(k_{20} - \frac{2}{7}k_{22}\right)\left(1 - \frac{5}{14}\epsilon^2\right)A_1' + \frac{1}{7}k_{31}\alpha'\beta^*B_6' \tag{5-78}$$

$$P_1 = \lambda^2 - \mu^2 \tag{5-79}$$

$$P_2' = \left(k_{20} - \frac{2}{7}k_{22}\right)\left(1 - \frac{5}{14}\epsilon^2\right)A_2' + \frac{1}{7}k_{31}\alpha'\beta^*B_7' \tag{5-80}$$

$$P_2 = \lambda\mu \tag{5-81}$$

$$P_3' = \left(k_{20} + \frac{1}{7}k_{22}\right)\left(1 - \frac{15}{14}\epsilon^2\right)A_3' - \frac{8}{7}k_{31}\alpha'\beta^*B_1' \tag{5-82}$$

$$P_3 = \lambda\nu \tag{5-83}$$

$$P_4' = \left(k_{20} + \frac{1}{7}k_{22}\right)\left(1 - \frac{15}{14}\epsilon^2\right)A_4' - \frac{8}{7}k_{31}\alpha'\beta^*B_2' \tag{5-84}$$

$$P_4 = \mu\nu \tag{5-85}$$

and

$$S'_1 = -\frac{1}{5}k_{21}A'_3 + k_{30}\alpha'\beta^*B'_1 \tag{5-86}$$

$$S_1 = \lambda(1 - 5\nu^2) \tag{5-87}$$

$$S'_2 = -\frac{1}{5}k_{21}A'_4 + k_{30}\alpha'\beta^*B'_2 \tag{5-88}$$

$$S_2 = \mu(1 - 5\nu^2) \tag{5-89}$$

$$S'_3 = \frac{3}{5}k_{21}A'_0 + k_{30}\alpha'\beta^*B'_3 \tag{5-90}$$

$$S_3 = \nu(3 - 5\nu^2) \tag{5-91}$$

$$S'_4 = k_{30}\alpha'\beta^*B'_4 \tag{5-92}$$

$$S_4 = \lambda(\lambda^2 - 3\mu^2) \tag{5-93}$$

$$S'_5 = k_{30}\alpha'\beta^*B'_5 \tag{5-94}$$

$$S_5 = \mu(3\lambda^2 - \mu^2) \tag{5-95}$$

$$S'_6 = k_{21}A'_1 + k_{30}\alpha'\beta^*B'_6 \tag{5-96}$$

$$S_6 = \nu(\lambda^2 - \mu^2) \tag{5-97}$$

$$S'_7 = k_{21}A'_2 + k_{30}\alpha'\beta^*B'_7 \tag{5-98}$$

$$S_7 = \lambda\mu\nu \tag{5-99}$$

and

$$T_1' = \left\{\frac{15}{14}\epsilon^2\left(k_{20} + \frac{1}{7}k_{22}\right) - \frac{9}{14}k_{22}\right\}A_3' + \frac{15}{7}k_{31}\alpha'\beta^*B_1' \tag{5-100}$$

$$T_1 = \lambda\nu\left(1 - \frac{7}{3}\nu^2\right) \tag{5-101}$$

$$T_2' = \left\{\frac{15}{14}\epsilon^2\left(k_{20} + \frac{1}{7}k_{22}\right) - \frac{9}{14}k_{22}\right\}A_4' + \frac{15}{7}k_{31}\alpha'\beta^*B_2' \tag{50102}$$

$$T_2 = \mu\nu\left(1 - \frac{7}{3}\nu^2\right) \tag{5-103}$$

$$T_3' = \left\{\frac{3}{!4}\left(k_{20} + \frac{2}{7}k_{22}\right)\epsilon^2 - \frac{9}{70}k_{22}\right\}A_0' - \frac{1}{7}k_{31}\alpha'\beta^*B_3' \tag{5-104}$$

$$T_3 = 3 - 30\nu^2 + 35\nu^4 \tag{5-105}$$

$$T_4' = k_{31}\alpha'\beta^*B_4' \tag{5-106}$$

$$T_4 = \lambda\nu(\lambda^2 - 3\mu^2) \tag{5-107}$$

$$T_5' = k_{31}\alpha'\beta^*B_5' \tag{5-108}$$

$$T_5 = \mu\nu(3\lambda^2 - \mu^2) \tag{5-109}$$

$$T_6' = \left\{\frac{5}{14}\left(k_{20} - \frac{2}{7}k_{22}\right)\epsilon^2 - \frac{3}{4}k_{22}\right\}A_1' - \frac{1}{7}k_{31}\alpha'\beta^*B_6' \tag{5-110}$$

$$T_6 = (\lambda^2 - \mu^2)(1 - 7\nu^2) \tag{5-111}$$

$$T_7' = \left\{\frac{5}{14}\epsilon^2\left(k_{20} - \frac{2}{7}k_{22}\right) - \frac{3}{4}k_{22}\right\}A_2' - \frac{1}{7}k_{31}\alpha'\beta^*B_7' \tag{5-112}$$

$$T_7 = \lambda\mu(1 - 7\nu^2) \tag{5-113}$$

Note that the components T_{x_i} of the perturbing acceleration depend, essentially, on the parameters

$$\lambda^*,\ \mu^*,\ \nu^*,\ r^*,\ a',\ m',\ x_1,\ x_2,\ x_3\ ,$$

$$T_{x_i} = T_{x_i}(\lambda^*, \mu^*, \nu^*, r^*, a', m', x_1, x_2, x_3)$$

To obtain the perturbing acceleration due to the moon tide, substitute for these parameters those specifying lunar position, orbital semimajor axis, and mass.

$$(T_{x_i})_L = T_{x_i}(\lambda^*_L, \mu^*_L, \nu^*_L, r^*_L, a'_L, m'_L, x_1, x_2, x_3)$$

Otherwise, substitute the solar parameters to produce the perturbing acceleration caused by the tidal action of the sun.

$$(T_{x_i})_S = T_{x_i}(\lambda^*_S, \mu^*_S, \nu^*_S, r^*_S, a'_S, m'_S, x_1, x_2, x_3)$$

COMPUTER PROGRAM OUTPUT

The inertial components $(T_{x_1})_L$, $(T_{x_2})_L$, $(T_{x_3})_L$, $(T_{x_1})_S$, $(T_{x_2})_S$, and $(T_{x_3})_S$ of the perturbing acceleration due to lunar and solar tides of the solid earth are input in terms of km sec^{-2}.

TRIAL DATA FOR PROGRAM CHECKOUT

The following trial case is restricted to the lunar tide. While executing this case, the entire algorithm was exercised and it was, thus, unnecessary to also consider the solar tide.

Input Data

$t \cdots J = 1977$		
$n = 88$		
$t^* = 57000$		sec
$\Delta t = 100$		sec
$\tilde{\omega} = 4.178074622E-03$		deg sec^{-1}

Epoch for the moon ephemeris (reference frame Σ_1)

$t_1 = 1950.0 = \mathrm{JD}2433282.423$

(1950.0 means beginning of Besselian year 1950)

Epoch for the satellite orbit (reference frame Σ_2)

$t_2 \cdots$	$J = 1977$	
	$n = 88$	
	$t^* = 0$	sec
t_2	$= \mathrm{JD}2443231.5$	

Moon coordinates, in frame Σ_1, at time $(t - \Delta t)$

x_L''	$= -\ 184258.5823$	km
y_L''	$= 329791.1351$	km
z_L''	$= 103840.2349$	km

Satellite position and velocity, in frame Σ_2, at time t

x_1	$= -\ 4009.582237$	km
x_2	$= +\ 103.9008135$	km
x_3	$= -\ 5269.570696$	km
$\dot{x}_1$	$= -\ 5.978210289$	km sec^{-1}
$\dot{x}_2$	$= +\ 1.710442216$	km sec^{-1}
$\dot{x}_3$	$= +\ 4.584971066$	km sec^{-1}
μ_E	$= 398601$	$km^3\ sec^{-2}$
R	$= 6378.145$	km
ϵ^2	$= 6.693421623E - 03$	
k_{20}	$= 0.3$	
k_{21}	$= 0.01$	
k_{30}	$= 0.1$	
k_{31}	$= 0.01$	
k_{22}	$= 0.1$	
m_L'	$= 7.3693281E + 22$	kg
M	$= 5.9731613E + 24$	kg
a_L'	$= 384400$	km

Results

θ = 0.1516444800 deg

ξ = 0.1744119454 deg

z = 0.1744282487 deg

T_0 = 0.5000000017 centuries

T = 0.2723967010 centuries

X_x = + 0.9999779632

Y_x = − 0.0060883617

Z_x = − 0.0026466801

X_y = + 0.0060883617

Y_y = + 0.9999814657

Z_y = − 0.0000080574

X_z = + 0.0026466801

Y_z = − 0.0000080567

Z_z = + 0.9999964975

x'_L = − 186537.2414 km

y'_L = + 328662.3531 km

z'_L = + 103349.5407 km

r^*_L = 391785.9267 km

$x_L^* = -188928.9046$ km

$y_L^* = +327293.3757$ km

$z_L^* = +103349.5407$ km

$\lambda^* = -0.4822248369$

$\mu^* = +0.8353882910$

$\nu^* = +0.2637908451$

$\lambda = -0.6054593779$

$\mu = +0.0156893457$

$\nu = -0.7957215507$

$r = 6622.380268$ km

$a = 6567.451187$ km

$K = 0.323073882E-05$ $km^2\ sec^{-2}$

$n = 0.118624368E-02$ sec^{-1}

$\alpha' = 0.1659246878E-01$

$\alpha = 0.9711750904$

$\beta^* = 0.9811480551$

$A'_0 = +0.1978107925$

$A'_1 = -0.3489996026$

$A'_2 = -0.1208534947E+01$

$A'_3 = -0.3816194918$

$A'_4 = +0.6611033498$

$B'_1 = -0.1179169837$

$B'_2 = +0.2042749770$

$B'_3 = +0.1748980753$

$B'_4 = +0.5609118737$

$B'_5 = -0.0001311647178$

$B'_6 = -0.4603145005$

$B'_7 = -0.1594002275E+01$

$P'_0 = +0.06443748370$

$P_0 = -0.5995183587$

$P'_1 = -0.09451271981$

$P_1 = +0.3663349027$

$P'_2 = -0.3272838250$

$P_2 = -0.009499261487$

$P'_3 = -0.1190554808$

$P_3 = +0.4817770751$

$P'_4 = +0.2062472668$

$P_4 = -0.01248435049$

$S'_1 = +0.0005712740433$

$S_1 = +0.1311342628E+01$

$S'_2 = -0.0009896538092$

$S_2 = -0.03398098796$

$S'_3 = +0.001471593023$

$S_3 = +0.1319815046$

$S'_4 = +0.0009131459348$

$S_4 = -0.2215028279$

$S'_5 = -0.2135318121E-06$

$S_5 = +0.0172503888$

$S'_6 = -0.004239372772$

$S_6 = -0.2915005769$

$S'_7 = -0.01468033233$

$S_7 = +0.007558767081$

$T'_1 = +0.02363141132$

$T_1 = -0.2300019019$

$T'_2 = -0.04093817407$

$T_2 = +0.005960068473$

$T'_3 = -0.002454126571$

$T_3 = -0.1963411384E+01$

$T'_4 = +0.9131459348E-04$

$T_4 = +0.1762545737$

$T'_5 = -0.2135318121E-07$

$T_5 = -0.01372650615$

$T'_6 = +0.02595922645$

$T_6 = -0.1257338135E+01$

$T'_7 = +0.08989303178$

$T_7 = +0.03260345555$

$V_0 = +1.0000000000$

$V_1 = +0.1742073243$

$V_2 = -0.1494101174$

$V_3 = +0.1899534786E-02$

$V_4 = -0.03055341082$

$C_0 = -0.0001622651725$ $km^3\ sec^{-2}$

$C_1 = +0.1353296915E+01$ $km^4\ sec^{-2}$

$C_2 = +0.8631523953E+06$ $km^5\ sec^{-2}$

$C_3 = +0.5505311134E+10$ $km^6\ sec^{-2}$

$C_4 = +0.3511367268E+14$ $km^7\ sec^{-2}$

$F = -0.007748690189$

$H = +0.1048876747$

$\rho_{11} = +0.2040473678$

$\rho_{12} = -0.004135328808$

$\rho_{13} = +0.1119464221$

$\rho_{21} = +0.03700735158$

$\rho_{22} = -0.004983233113$

$$\rho_{23} = +0.1740434693$$

$$\rho_{31} = +0.3829649087$$

$$\rho_{32} = -0.01385120787$$

$$\rho_{33} = +0.2036553308$$

$$T_{x_1} = -0.1929747594E-09 \quad \text{km sec}^{-2}$$

$$T_{x_2} = +0.9604765343E-10 \quad \text{km sec}^{-2}$$

$$T_{x_3} = -0.1824727285E-09 \quad \text{km sec}^{-2}$$

$$T = 0.2824193799E-09 \quad \text{km sec}^{-2}$$

DEVELOPMENT STATUS OF THE COMPUTER ROUTINES

By early summer 1978, all five tide routines had been coded and made part, as operational subroutines, of the TERRA and CELEST computer program systems. Program checkout was accomplished (separately for each tide routine) by first selecting typical input data that would exercise the algorithm to a sufficient degree. Subsequently, the programmer would run the algorithm under test on the computer, while the author would perform identical calculations on a programmable electronic calculator. Agreement to ten significant digits of the computer results with those of the manual computation was taken as evidence for successful computer implementation of the algorithm.

As indicated by the last paragraph of the INTRODUCTION, the two ocean tide algorithms may be expected to produce nearly identical perturbing accelerations because the tidal ocean surface of the second algorithm is a functionalization of the discrete representation of the ocean surface on which the first algorithm is based. Like the routines for the two air tides and that for the solid earth tide, each of the two ocean tides had been checked out individually. In the process, to facilitate the manual calculations, a simple ocean surface had to be selected for each of the two algorithms. Because of the different mathematical structure of the two algorithms, simplicity meant that each algorithm required its own individual surface. Thus, it was impossible to directly compare the resulting perturbing accelerations. This in turn meant that, during this phase of the checkout, the

mutual consistency of the two ocean tide routines could not be verified. Because the two routines are so complex, this remained a source of concern despite the caution practiced while deriving them. The question was finally settled by running each algorithm, using the complete input data base. The resulting perturbing accelerations agreed with each other to 20 percent. Considering the differences in the physical definitions of the tidal mass layers (one being a discrete point set, and the other being a continuous surface), this was interpreted as satisfactory evidence for the validity of the algorithms.

For operational use, that ocean tide routine was selected which employs the discrete tide surface representation. Realizing that the second algorithm contains a more accurate physical model (by virtue of its continuous tidal mass layer as compared with the point mass approximation inherent in the first ocean tide routine), it was yet considered a decisive advantage of the algorithm chosen that it is capable of accepting among its input, and of being capable to directly process, without the need for any intervening computational steps, the earlier mentioned tide amplitude and phase tables that are in-house products of NSWC. This aspect is particularly important because it is expected that, during the useful life of TERRA/CELEST, the present M_2 tide tables will be augmented by similar tables for various other constituents of the ocean tide spectrum. Once available, a mere extension of the present algorithm will make possible the immediate use of the additional program input, without first having to obtain the additional functionalizations of the tidal ocean that would be needed if the second ocean tide routine had been adopted for inclusion in TERRA/CELEST.

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APPENDIX A

NOTES CONCERNING THE SECOND OCEAN TIDE ROUTINE

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As mentioned in the INTRODUCTION, the second computer routine for the ocean tide is based on a functionalization of the tidal ocean surface. This functionalization represents the height of the tidal surface above mean sea level, h, as an expansion in surface harmonics.

$$h = \zeta \cos(\sigma t^* + \chi - \delta) = \zeta \cos\delta \cos(\sigma t^* + \chi) + \zeta \sin\delta \sin(\sigma t^* + \chi) \tag{A-1}$$

$$\zeta \cos\delta = \sum_{n=0}^{15} \sum_{m=0}^{n} (C_{nm} \cos m\lambda + S_{nm} \sin m\lambda) P_n^m (\sin\theta) \tag{A-2}$$

$$\zeta \sin\delta = \sum_{n=0}^{15} \sum_{m=0}^{n} (C'_{nm} \cos m\lambda + S'_{nm} \sin m\lambda) P_n^m (\sin\theta) \tag{A-3}$$

where

λ = longitude
θ = latitude
C_{nm}, S_{nm}, C'_{nm}, S'_{nm} = expansion coefficients
σ = rate of the mean mean longitude of the moon
χ = mean mean longitude of the moon, at the beginning of the day UT

Note that h is essentially a function of longitude, latitude, and time. This function is valid everywhere on the globe (sea and land – see the comments at the end of this appendix).

The surface density corresponding to the tidal mass layer is

$$s = \rho h = \frac{1}{G}\left\{\alpha \cos(\sigma t^* + \chi) + \beta \sin(\sigma t^* + \chi)\right\} \tag{A-4}$$

$$\alpha = G\rho\ \zeta \cos\delta \tag{A-5}$$

$$\beta = G\rho\ \zeta \sin\delta \tag{A-6}$$

The gravitational potential associated with the tidal bulge is then

$$\phi = G\oint \frac{s\, dS'}{|\bar{r} - \bar{r}'|} = \phi_1 \cos(\sigma t^* + \chi) + \phi_2 \sin(\sigma t^* + \chi) \tag{A-7}$$

$$\phi_1 = \oint \frac{\alpha\, dS'}{|\bar{r} - \bar{r}'|} \tag{A-8}$$

$$\phi_2 = \oint \frac{\beta\, dS'}{|\bar{r} - \bar{r}'|} \tag{A-9}$$

$$\oint dS' = R^2 \int_0^{2\pi} d\lambda' \int_{-\pi/2}^{+\pi/2} \cos\theta'\, d\theta' \tag{A-10}$$

$$\frac{1}{|\bar{r} - \bar{r}'|} = \sum_{n=0}^{\infty} \frac{R^n}{r^{n+1}} \Big\{ P_n(\sin\theta)\ P_n(\sin\theta')$$

$$+ 2\sum_{m=1}^{n} \frac{(n-m)!}{(n+m)!} P_n^m(\sin\theta)\ P_n^m(\sin\theta')\ \cos m(\lambda - \lambda') \Big\} \tag{A-11}$$

Now, evaluate ϕ_1

$$\phi_1 = G\rho R^2 \int_0^{2\pi} d\lambda' \int_{-\pi/2}^{+\pi/2} \cos\theta'\, d\theta' \left\{ \frac{\zeta \cos\delta}{|\bar{r} - \bar{r}'|} \right\} \tag{A-12}$$

Let

$$\sin\theta = y \tag{A-13}$$

$$\sin\theta' = y' \tag{A-14}$$

and consider the term ij from the expansion for $\zeta \cos \delta$, namely

$$(\zeta \cos \delta)_{ij} = (C_{ij} \cos j\lambda' + S_{ij} \sin j\lambda')P_i^j(y') \tag{A-15}$$

Then,

$$\begin{aligned}
(\phi_1)_{ij} = {} & G\rho R^2 C_{ij} \int_0^{2\pi} d\lambda' \int_{-1}^{+1} dy' \; P_i^j(y') \cos j\lambda' \\
& * \left\{ \sum_{n=0}^{\infty} \frac{R^n}{r^{n+1}} \left[P_n(y) \; P_n(y') \right.\right. \\
& \left.\left. + 2\sum_{m=1}^{\infty} \frac{(n-m)!}{(n+m)!} P_n^m(y) \; P_n^m(y') \cos m(\lambda - \lambda') \right] \right\} \\
& + G\rho R^2 S_{ij} \int_0^{2\pi} d\lambda' \int_{-1}^{+1} dy' \; P_i^j(y') \sin j\lambda' \\
& * \left\{ \sum_{n=0}^{\infty} \frac{R^n}{r^{n+1}} \left[P_n(y) \; P_n(y') \right.\right. \\
& \left.\left. + 2\sum_{m=1}^{\infty} \frac{(n-m)!}{(n+m)!} \; P_n^m(y) \; P_n^m(y') \cos m(\lambda - \lambda') \right] \right\}
\end{aligned} \tag{A-16}$$

Repeated application of the orthogonality relationships for the Legendre functions, as well as for the sine and cosine functions, makes possible the following equations:

$$\begin{aligned}
(\phi_1)_{i,j=0} & = G\rho R^2 C_{io} 2\pi \int_{-1}^{+1} dy' \; P_i(y') \sum_{n=0}^{\infty} \frac{R^n}{r^{n+1}} \; P_n(y) \; P_n(y') \\
& = 2\pi R^2 G\rho \frac{2}{2i+1} \frac{R^i}{r^{i+1}} C_{io} \; P_i(y)
\end{aligned} \tag{A-17}$$

$$(\phi_1)_{i,j\neq 0} = 2G\rho R^2 C_{ij} \sum_{n=0}^{\infty} \sum_{m=1}^{n} \left[\frac{(n-m)!}{(n+m)!} \frac{R^n}{r^{n+1}} \int_{-1}^{+1} P_i^j(y') \, P_n^m(y') \, dy' \right.$$

$$\left. * \; P_n^m(y) \int_0^{2\pi} \cos j\lambda' \, \cos m(\lambda - \lambda') \, d\lambda' \right]$$

$$+ \; 2G\rho R^2 S_{ij} \sum_{n=0}^{\infty} \sum_{m=1}^{n} \left[\frac{(n-m)!}{(n+m)!} \frac{R^n}{r^{n+1}} \int_{-1}^{+1} P_i^j(y') \, P_n^m(y') \, dy' \right.$$

$$\left. * \; P_n^m(y) \int_0^{2\pi} \sin j\lambda' \, \cos m(\lambda - \lambda') \, d\lambda' \right] \qquad \text{(A-18)}$$

$$\int_0^{2\pi} \cos j\lambda' \, \cos m(\lambda - \lambda') \, d\lambda' = \pi \, \delta_{mj} \, \cos j\lambda \qquad \text{(A-19)}$$

$$\int_0^{2\pi} \sin j\lambda' \, \cos m(\lambda - \lambda') \, d\lambda' = \pi \, \delta_{mj} \, \sin j\lambda \qquad \text{(A-20)}$$

$$\delta_{nm} = \begin{cases} 1 & \text{if} \quad n = m \\ 0 & \phantom{\text{if}} \quad n \neq m \end{cases} \qquad \text{(A-21)}$$

$$(\phi_1)_{i,j\neq 0} = 2\pi G\rho R^2 \frac{2}{2i+1} \frac{R^i}{r^{i+1}} \; P_i^j(\sin\theta) * (C_{ij} \cos j\lambda + S_{ij} \sin j\lambda) \qquad \text{(A-22)}$$

and consequently,

$$(\phi_1)_{ij} = 2\pi R^2 G\rho \frac{2}{2i+1} \frac{R^i}{r^{i+1}} * (C_{ij} \cos j\lambda + S_{ij} \sin j\lambda) \; P_i^j(\sin\theta) \qquad \text{(A-23)}$$

Thus, the following results in a similar fashion:

$$(\phi_2)_{ij} = 2\pi R^2 G\rho \frac{2}{2i+1} \frac{R^i}{r^{i+1}} * (C'_{ij} \cos j\lambda + S'_{ij} \sin j\lambda) \; P_i^j(\sin\theta) \qquad \text{(A-24)}$$

Noting that the original expansion of the tidal surface is limited to $n_{max} = 15$, the tide potential is now

$$\phi = \sum_{\nu=0}^{15} \sum_{\mu=0}^{\nu} \left[F_{\nu\mu} U_{\nu\mu} + H_{\nu\mu} V_{\nu\mu}\right] \tag{A-25}$$

$$F_{\nu\mu} = \frac{2\pi R^2 G\rho}{\mu_E} \frac{2}{2\nu + 1} \left[C_{\nu\mu} \cos(\sigma t^* + \chi) + C'_{\nu\mu} \sin(\sigma t^* + \chi)\right] \tag{A-26}$$

$$H_{\nu\mu} = \frac{2\pi R^2 G\rho}{\mu_E} \frac{2}{2\nu + 1} \left[S_{\nu\mu} \cos(\sigma t^* + \chi) + S'_{\nu\mu} \sin(\sigma t^* + \chi)\right] \tag{A-27}$$

$$U_{\nu\mu} = \mu_E \frac{R^\nu}{r^{\nu+1}} P_\nu^\mu (\sin\theta) \cos\mu\lambda \tag{A-28}$$

$$V_{\nu\mu} = \mu_E \frac{R^\nu}{r^{\nu+1}} P_\nu^\mu (\sin\theta) \sin\mu\lambda \tag{A-29}$$

Up to here, each of the physical quantities occurring in the equations may still be entered in terms of arbitrary units. However, for the purpose of the second ocean tide computer algorithm, it is necessary to employ compatible units: kilometers for length, kilograms for mass, and mean solar seconds for time. As obtained from NOAA, the coefficients $C_{\nu\mu}$, $S_{\nu\mu}$, $C'_{\nu\mu}$, and $S'_{\nu\mu}$ are listed in meters. To accommodate the existing computer card deck for these data, it is necessary to multiply the right sides of Equations A-26 and A-27 by 10^{-3} prior to their use in the main part of this report.

It should finally be mentioned that the expansion for the tidal surface (Equations A-1 through A-3) is intended to be valid for both sea and land in the sense that it is a fit to the NSWC/Schwiderski tidal heights and phases on the ocean and to zero tidal heights on land. Because of the relatively limited number of terms employed in the expansion, this fit is somewhat imperfect, especially near the coast lines. This imperfection is intentional. The reason for it is that the expansion was originally meant to be the basis for finding a tide potential for satellite work rather than a continuous, highly accurate description of the tidal surface. Satellite orbits appear to be much more sensitive to the low-order harmonics in the tidal disturbing potential than to the higher ones. Based on information from an outside source, we in fact anticipate that certain of the second- and fourth-order harmonics will be the main contributors to the orbital perturbation. Thus, the emphasis on the low-order terms apparent in the expansion for the tidal surface is rather an advantage than a defect as far as our own satellite work is concerned.

APPENDIX B

A COMPARISON OF THE PERTURBING ACCELERATION DUE TO THE SOLID EARTH TIDE WITH THAT CAUSED BY THE OCEAN TIDE

A COMPARISON OF THE PERTURBING ACCELERATION DUE TO THE SOLID EARTH TIDE WITH THAT CAUSED BY THE OCEAN TIDE

Preliminary runs of the first computer routine for the ocean tide and of the routine for the solid earth tide, all of which used complete sets of input data (as opposed to the restricted input data of the numerical trial cases listed in the main body of this report), have resulted in the following perturbing accelerations.

The satellite coordinates and the perturbing acceleration components listed are referred to the inertial frame corresponding to the mean equinox and mean equator of J = 1977, n = 202, and t* = 0 sec.

Time t

$$\begin{aligned} J &= 1977 \\ n &= 202 \\ t^* &= 50000 \text{ sec} \end{aligned}$$

Satellite coordinates at t

$$\begin{aligned} x_1 &= 3151.52923 \text{ km} \\ x_2 &= 5458.60875 \\ x_3 &= 3639.07250 \end{aligned}$$

Perturbing acceleration due to ocean tide at t

$$\begin{aligned} T_x &= +0.594629\text{E}-11 \text{ km sec}^{-2} \\ T_y &= +0.134622\text{E}-10 \\ T_z &= -0.258193\text{E}-11 \end{aligned}$$

Perturbing acceleration due to solid earth tide at t

$$\left.\begin{array}{l} \left.\begin{aligned} T_x &= +0.149368\text{E}-09 \text{ km sec}^{-2} \\ T_y &= +0.125475\text{E}-10 \\ T_z &= -0.184573\text{E}-10 \end{aligned}\right\} \text{due to moon} \\ \\ \left.\begin{aligned} T_x &= -0.493523\text{E}-10 \\ T_y &= +0.381372\text{E}-10 \\ T_z &= +0.713397\text{E}-12 \end{aligned}\right\} \text{due to sun} \end{array}\right\} \begin{array}{c}\text{Nominal values for} \\ \text{Love numbers}\end{array}$$

$$\left.\begin{array}{l} T_x = +0.164304E-09 \\ T_y = +0.013802E-09 \\ T_z = -0.020303E-09 \end{array}\right\} \text{due to moon}$$

$$\left.\begin{array}{l} T_x = -0.054288E-09 \\ T_y = +0.041951E-09 \\ T_z = +0.000785E-09 \end{array}\right\} \text{due to sun}$$

Each Love number increased by 10 percent

$$\left.\begin{array}{l} T_x = +0.179241E-09 \\ T_y = +0.150570E-10 \\ T_z = -0.221487E-10 \end{array}\right\} \text{due to moon}$$

$$\left.\begin{array}{l} T_x = -0.592228E-10 \\ T_y = +0.457647E-10 \\ T_z = +0.856076E-12 \end{array}\right\} \text{due to sun}$$

Each Love number increased by 20 percent

The magnitudes of the perturbing accelerations are then

T(Ocean tide) = T_{OCEAN} = 1.49E - 11 km sec^{-2}

T(Solid earth tide due to moon, for nominal Love Numbers) = T_{SOLID} = 1.51026E - 10 km sec^{-2}

T(Solid earth tide due to moon, for Love numbers increased by 10 percent) = $T_{SOLID+10}$ = 1.66048E - 10 km sec^{-2}

T(Solid earth tide due to moon, for Love numbers increased by 20 percent) = $T_{SOLID+20}$ = 1.81231E - 10 km sec^{-2}

Note that

$$T_{SOLID+10} - T_{SOLID} \approx T_{OCEAN} \qquad \text{(B-1)}$$

or

$$|\overline{T}_{SOLID+10} - \overline{T}_{SOLID}| \approx |\overline{T}_{OCEAN}| \qquad \text{(B-2)}$$

This indicates that, for the particular combination of time and satellite position chosen, if one increases the Love numbers by ten percent, the perturbing acceleration due to the lunar component of the solid earth tide increases by an amount roughly equal to the perturbing acceleration due to the ocean tide. For the particular example chosen, the two equations are accurate to better than two percent, which is suspected to be a coincidence. But we are certain that this example reveals the order of magnitude of a relationship existing between the gravitational effects of the ocean tide and the solid earth tide.

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